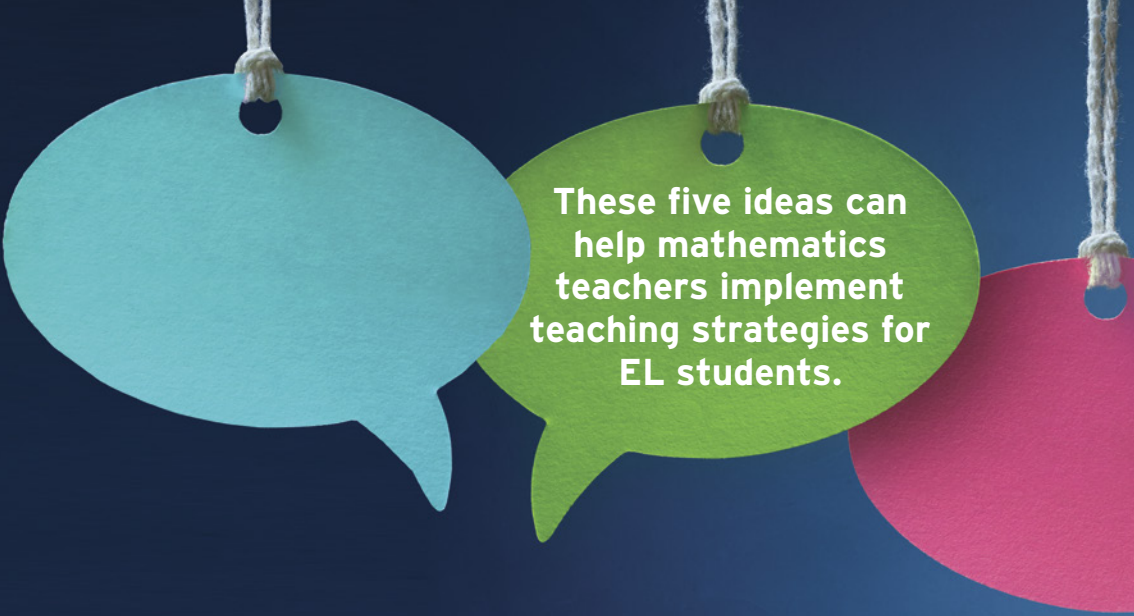




These five ideas can help mathematics teachers implement teaching strategies for EL students.

SUPPORTING ENGLISH LEARNERS IN THE MATH CLASSROOM:

Five

Have you ever presented a mathematics problem to your students and gotten blank stares? Have your students told you that they do not understand because they do not speak English? After years of supporting English learners (ELs) in high school mathematics classes, we have put together some strategies that have proven beneficial to support students' math skills and language. In a typical mathematics class in a district near the authors, more than 30 percent of students are learning English, and although some of them are fluent and some are bilingual, others are just beginning to learn English.

According to the National Center for Education Statistics (2016), the number of public school students in the United States who were ELs was estimated at 4.5 million. However, in 2005, the national standardized testing scores (Fry 2007) show that about 51 percent of eighth-grade EL students were behind other students in reading and math. In our district, this is true as well.

Many teachers are trained to be mathematics teachers in a setting where students speak only English. However, with the growing number of EL students around the country, teaching mathematics now includes teaching language as well. This can be overwhelming for novice teachers.

Tools exist that can increase English language proficiency and content knowledge. The Sheltered Instruction Observation Protocol (SIOP) provides

structure for teachers to plan lessons that meet the linguistic and content needs of students who are learning English. The SIOP informed our implementation of the mathematics tools included in this work (Echevarria, Vogt, and Short 2013). Our five tools are based on the eight components of the SIOP model. In general, these tools allow teachers to get to know their students and their prior knowledge, build background, and most important, develop their math and English language with strategies, lesson planning that includes content and language objectives, interaction, and comprehensible input. These tools also aim to narrow the gap between different cultures, increase students' mathematics vocabulary, promote motivation and understanding when engaging with word problems, develop writing skills, and improve oral expression ability for EL students in the mathematics classroom.

TOOL 1: BUILDING CULTURAL BACKGROUND KNOWLEDGE

Why do we need to build cultural background knowledge in mathematics? Research indicates that some teachers see math as a universal language, and there is a never-ending debate about whether mathematics has its own language. Given the computational/procedural nature of mathematics, formulations and Arabic numerals have been widely and internationally used (Rolka 2004). However, more and more studies show that math has its own language, which researchers

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Useful Tools

call *math register*. Moschkovich (2007) suggests that mathematical registers are situated within a culture, community, or group that has informally developed this register. Pimm (1987) indicates that mathematics is a language with its own system of meaning making and construction that might be different from everyday speech.

Not only does mathematics have its own register, but it also has “dialects.” According to Dong (2004, p. 73), different cultures have different ways to solve division problems (see **fig. 1**). Without awareness of cultural differences, some teachers might identify EL students as having limited mathematics knowledge; however, the truth is that they know how to do mathematics, they just do not know how to do it “our way.” Acknowledging different methods of solving problems is important, especially when students understand in a systematic way how to get the correct answer. Solving mathematics problems in a different way should not trigger a penalty.

Therefore, doing some background research on our ELs before we start a new unit is necessary. We ought to take notice of the difference in mathematical language and work with these students to help them bridge our mathematical dialect with their own. As we see from **figure 1**, making any assumptions about what students know on the basis of how they solve a problem is risky. Even with just a small difference in expression, misdiagnosis of ELs’ mathematics aptitude can

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China, Japan, Korea, U.S.	Puerto Rico
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Vietnam	Poland
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Colombia	Laos

Fig. 1 Students may use several different ways to solve division problems.



occur. Sometimes the information in a student's particular mathematics dialect is limited. To build prior knowledge about their ELs, teachers can always find answers directly from their students.

Figure 2 shows examples of questions posed to collect data on students' prior knowledge. This can be done using students' primary languages and can be shared among groups of ELs. For example, Portuguese and Spanish speakers will find that the word for *square* is pronounced the same in both languages (*cuadrado* in Spanish and *quadrado* in Portuguese).

With these pre-unit questions, we can quickly adjust our lesson plan according to our students' need for improvement. If most students are using centimeters (cm) when they describe a length, then we should be very cautious about the units we use to describe different sizes of shapes. When asked, for example, "How many *dimes* do you think can fit inside a 2-by-2 inch square?" students who are familiar with *cm* rather than *inches* might struggle with understanding just how big a 2-by-2 inch square is. The preteaching for this might be to build a ruler to scale with centimeters and corresponding inches for them to use during math practice. Additionally, students from other countries may not know the names of U.S. coins, but they may know the value. So, before doing any lesson with money, discuss that a dime equals ten cents. In Spanish, *dime* means "tell me."

TOOL 2: BUILDING VOCABULARY IN MATHEMATICS

Many mathematics teachers might assume that language and math register can only be learned separately and are independent of each other. This belief is based on assumptions that students will not be able to engage in the classroom and teachers will not be able to teach mathematics

Example of Questions

1. What actions do you take when solving an equation or inequality?

¿Cuales acciones tomas cuando estás resolviendo una ecuación o desigualdad?

在解决计算公式问题的时候你一般用什么方法/什么步骤?

2. What words do you know for shapes and what objects are examples of those shapes?

For example: sphere=🍷

¿Cuales palabras sabes por figuras y cuales objetos son ejemplos de estas figuras?

Ejemplo: esfera=🍷

你知道的哪一些形状的名称, 你能给出与形状相应的例子吗?

例子: 球体=🍷

3. How do you use Measurement outside of school? In what units do you measure?

¿Cómo utilices medidas fuera de la escuela? ¿Cuales unidades usas?

你在学校之外是如何应用测量的? 你用什么测量单位?

unless the student is English proficient (Fernandes 2012). Students in mathematics classes can benefit from the attention to language objectives as well as content objectives. Students must "develop and strengthen the language and literacy skills needed for academic success" (Echevarria, Vogt, and Short 2013, p. 38). Research shows that learning languages can help students develop their metacognitive skills for information processing. In East Asian languages, the structure of the base-ten number system is transparently represented in the structure of the number words themselves. For instance, the Chinese number words for 11, 12, and 13 are translated as *ten one*, *ten two*, and *ten three*, respectively. (Geary et al. 1996). In French, 90 is written as *quatre-vingt-dix*, which can be interpreted as $4 \cdot 20 + 10$. According to Bialystok (1999), code-switching between different languages can also help bilingual students to learn and show cognitive development in their number sense and cardinal number knowledge. Mathematics teachers can wisely use this as an advantage when teaching math vocabulary.

The purpose of the activity shown in **figs. 3** and **4** is to help students find connections between terms and attributes. For instance, in Chinese *tri-angle* is translated as

“三 (three) 角 (angles) 形 (shape).”

Students might conclude that a triangle has three angles. Similarly in French, Spanish, Italian, and Portuguese as well as English, *tri-* means *three*. If you use visual examples of a tricycle, triple, and trio, students can determine on their own what the prefix *tri-* means, as those are cognates in the respective languages. Then, angle is also a cognate, and they can infer the meaning of a triangle even if they do not know the direct translation of the word. When beginning this work, we cannot expect every student, including native speakers, to be able to break down all words linguistically, but if they are guided to find cognates or words that look like those they already know in their dominant languages, then they can often understand the new words and be able to use this strategy in other academic areas. Short and Echevarría (2013, p. 68) state that "effective language learners make use of their native language and literacy skills" by identifying cognates.

TOOL 3: SUPPORTING READING: RELATED TEXT IN MATHEMATICS WORD PROBLEMS

Word problems contain many abstract mathematics concepts and terms. These problems are not presented as a narrative, as one would read in an English class, but as a descriptive puzzle that

Fig. 2 Collecting data on prior knowledge can be done in the student's native language

students need to decode. When word problems are not engaging, students can lose interest and focus. For EL students, decoding a traditional word problem can be difficult, particularly when they are not invested in the story or do not understand key words or even names. One way to maintain interest is to use names of students in the class and use situations that are culturally, linguistically, and developmentally appropriate. Below are examples.

Kiara, Ana, and Ivette are playing soccer this season. Kiara scored 3 times as many goals as Ivette. Ivette scored 7 more goals than Ana. The total number of goals scored is 38. How many goals did each girl score?

Kiara, Ana, e Ivette están jugando fútbol esta temporada. Kiara marcó tres veces más goles que Ivette. Ivette marcó 7 goles más que Ana. El total de goles marcados es 38. ¿Cuántos goles marcó cada niña?

Researchers (Chappell and Thompson 1999) have shown that other reading materials, such as news reports and stories, can be used as tools to teach abstract mathematics concepts and promote interest and motivation in learning mathematics. *The Crest of the Peacock: Non-European Roots of Mathematics*, written by George Joseph (2010), discusses mathematics history from Egypt, Babylonia, Ancient China, and Ancient India. Joseph cites the ancient Chinese as having discovered the relationship among the three sides of a right triangle. In *Jiu Zhang*, also known as the *Mathematical Treatise in Nine Sections*, the last chapter is called the *Gou Gu*, or *Base-Height*. In the Gou Gu theorem, the shorter leg of a right triangle is the gou, the longer leg the gu, and the xian is the hypotenuse (see fig. 5).

If we can use this Gou 3, Gu 4, and Xian 5 to determine the relationships among the three sides in a right-angled triangle, Chinese EL students will become engaged, and other students with a different cultural background might want to learn the history behind the equation.

TOOL 4: SUPPORTING WRITING IN MATHEMATICS

The appearance of writing now in the Common Core State Standards for Mathematics has prompted an increased emphasis on writing in the math classroom. Today's students must be able to solve problems and must be able to show their thinking step by step. A double-entry journal is one way to encourage students to show their thinking. Research findings have demonstrated repeatedly that double-entry journals provide a medium for English learners to record, communicate, and

	Triangle	Octagon
Name of shape in your language	三角形	八角形
Attributes	3 条边, 3 个角 3 lines, 3 angles 180° and 180°	8 lados iguales 8 equal sides
Picture		
Real-life object		
Sentence in English	The three teammates pass the ball in a triangle.	I don't like to sit at octagonal tables.

Fig. 3 Students can extend their mathematics vocabulary and academic language by making connections between terms and attributes.

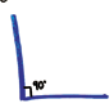
ecuación cuadrática $ax^2 + bx + c = 0$ Quadratic Equation	Cubo  Cube	Desigualdad $x + 2 > 5$ $2x < 6$ Inequality
Teorema de Pitágoras $a^2 + b^2 = c^2$  Pythagorean Theorem	Variable $\square + 9$ $25 \div y$ $7x$ $a + 10$ Variable	Ángulo recto  Right Angle

Fig. 4 As in figure 3, connections are made that grow student's mathematics vocabulary.

reflect on mathematics using the written language (McIntosh and Draper 2001)

Figure 6 is an example of a double-entry journal. We encourage EL students to use their native language as a mediator between the math problem and the explanation of their thinking in English. Doing this writing during math time, students can use vocabulary and write different kinds of text while practicing their mathematics concepts. Students can be asked to post a tweet (140 characters) to explain the definition of hypotenuse. They can also write a short story using math vocabulary, making sure that they use the correct math dialect. For example, students can choose ten words from their mathematics vocabulary and write how they solved a problem or use the same words in a piece that is completely fictional.

The teacher can ask how students used mathematics the previous day as part of a journal

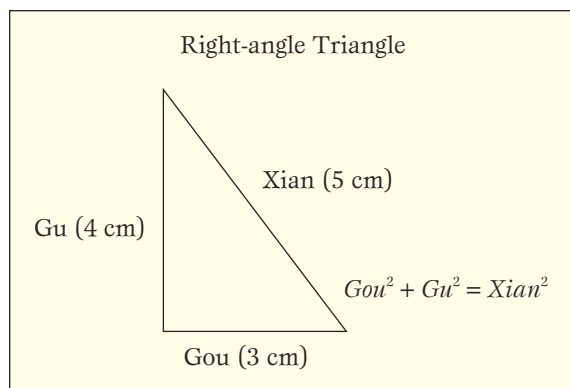


Fig. 5 The Gou-Gu theorem is used to find missing sides of right triangles.

writing activity. Some answers might involve counting change, calculating how much time remained in a soccer game, measuring ingredients while preparing dinner, and calculating a grade, a GPA, or a scoring average for basketball.

TOOL 5: SUPPORTING SPEAKING: ENCOURAGING A “FREE” LANGUAGE ENVIRONMENT

Mathematics teachers might notice that some EL students seem to be quiet during class discussions and more active when they are assigned individual work. According to Moschkovich (2007), a teacher might attribute slight delays in responding orally or longer delays on written assessment to students not knowing their math facts. However, not knowing math facts might not be the reason for their silence. Research shows that students who profit from certain opportunities are not always the ones with more mathematical knowledge than other students, but instead those whose dominant language and participation are prioritized in group interaction (Planas 2014). Seeing native English-speaking students taking charge of group discussions in class is not surprising; they feel comfortable using English to participate in group interactions. In addition to this, a study completed by Marsh and Maki in 1976 showed that bilingual adults performed arithmetic operations more rapidly in their preferred language than in their non-preferred language and that switching from one language to another during an experimental session slowed reaction times. If students are asked to solve a math problem by using their second language, it might take longer than using their preferred language. If they seem to be quiet during class, it might be because they need more time to process. Teachers can build a “free” language environment for those EL students who feel more comfortable doing mathematics in their native language. Moschkovich (2007) suggests that teachers should allow bilingual learners to choose the language for carrying out their own arithmetic computations and to take more time on timed

Math Problem	Thinking Process
Kiara 3 times Ivette Ivette 1 more Ana Total is 38 Each girl ?	I put Kiara as X because they are to relate to each other. So Ivette = $\frac{X}{3}$ because Kiara is 3 times as many as Ivette. Then Ana = $\frac{X}{3} + 1$ because Ivette is scored 1 more than Ana. So $X + \frac{X}{3} + \frac{X}{3} + 1 = 38$ $X + \frac{2X}{3} = 45$ $3(X + \frac{2X}{3}) = 45 \times 3$ $3X + 2X = 45 \times 3$ $5X = 135$ $X = 27$ Kiara score 27, Ivette score $\frac{27}{3} = 9$, Ana score $9 + 1 = 10$.

Fig. 6 A Double-Entry Journal allows students to share their answers with the group.

tests of arithmetic computation. Three steps can create such a “free” classroom environment:

1. Class warm-up: Give students a mathematics problem to show what they have already learned.
2. Individual problem solving: Give students five to eight minutes to solve the problem.
3. Group discussion: Ask students to use a double-entry journal and have them share their solution in their group (see **fig. 6**).

For EL students, using their native language should always be acceptable for helping them solve a problem, as long as they can be given feedback so they can understand. Students can also use visual solutions to solve problems when they are unsure how to express what they have done to solve the problem at hand. Many students often benefit from being able to share at their table, in a small, linguistically homogeneous group strategically organized with a bilingual student to help interpret the findings.

These five tools cover ideas to support EL students as they learn mathematical vocabulary, and to read, write, and speak mathematics. Moreover, these tools support teachers as they create a multicultural and welcoming learning environment. They provide an important bridge for every mathematics teacher with his or her EL students, helping to infuse language learning into mainstream classrooms. Teachers can pack these tools in their virtual toolkit and step out, taking their first steps in the classroom with confidence.

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Internet Resources to Support English Learners in Math

Instructional Materials and Glossaries from NYU/Steinhardt

<http://steinhardt.nyu.edu/metrocenter/resources/glossaries>

This resource offers bilingual glossary support in multiple languages in multiple content areas and grade levels. The glossaries are approved by the State of New York (whereas the instructional materials are specifically for instruction).

Multicultural Math: Lessons from the Mayas from the NEA

<http://www.nea.org/tohtols/lessons/47756.htm>

You can find lesson plans, activities, background resources, videos, downloadables, and online resources. A cultural resource for students from southern Mexico, Guatemala, northern Belize, and western Honduras.

From the Math Forum, a Collection of Multicultural Resources

http://mathforum.org/library/ed_topics/multiculturalism/

More than twenty different ethnomathematics sites that address mathematical resources for students of different cultures

The Story of Mathematics

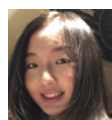
<http://www.storyofmathematics.com/story.html>

"Follow the story as it unfolds in this series of linked sections, like the chapters of a book. Read the human stories behind the innovations, and how they made—and sometimes destroyed—the men and women who devoted their lives to the story of mathematics."

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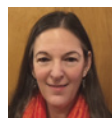
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