Why is it so crucial that coaches and teachers concentrate their interactions on students' mathematical reasoning?

Coaching through Focusing on Student Thinking

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Imagine that you are a school-based mathematics coach whose goal is to support teachers in developing and refining their teaching so that students have opportunities to build identities as mathematical thinkers and engage deeply in the Common Core's (CCSSI 2010) Standards for Mathematical Practice (SMP). As you walk into teachers' classrooms to support them, you consider the following questions:

- Which developments will I look for as the lesson plays out?
- Which conversations might I have with teachers during or following the lesson?
- Which conversations do I want to have with teachers across a grade level during collaborative planning time?

You understand that the content of those conversations is of utmost importance, especially given the current urgency to help teachers develop within their classrooms rich discourse communities in which students' ideas are valued. This article focuses on productive conversations that mathematics coaches can conduct with teachers.

Curiosity about student thinking lies at the heart of great mathematics teaching. When teachers are curious about and grounded in their students' thinking, their proficiency at eliciting, interpreting, and responding to student ideas increases (NCTM 2014). Coaching conversations that focus on students' thinking can support teachers in developing their instructional practices. When coaches frame their interactions with teachers by first examining students' thinking *together*, they can then problem solve together about next steps in instruction and engage in rich conversations about teaching and learning mathematics. In this article, we consider two related coaching conversations that arose from examining students' thinking during one-on-one coaching and during teacher collaborative time.

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Developing classrooms where student thinking is valued

For the last five years, Tara, a mathematics coach at Hilltop, an urban elementary school, has been working alongside all teachers at her K–grade 5 school to enact a vision of teaching that focuses on promoting reasoning and sense making. Tara strives to support teachers in enacting a vision of teaching similar to that described in *Principles to Actions: Ensuring Mathematical Success for All* (NCTM 2014). In this article, we will examine how she helps a group of fourth-grade teachers use—

> evidence of student thinking to assess progress toward mathematical understanding and to adjust instruction continually in ways that support and extend learning. (NCTM 2014, p. 10)

We look at two coaching conversations in which Tara supports the fourth-grade teachers as they implement a fractions unit.

Classroom coaching conversations

Let's drop in to a classroom to see how the coach enters a coaching conversation with the teacher through examining students' work. We first visit Soren's fourth-grade classroom, where students are working in pairs on a set of problems that give them opportunities to add or subtract mixed numbers with like denominators (see **fig. 1**). The mathematics coach developed the problems—addition and subtraction of mixed fractions with like denominators—on the basis of what she had learned about sequencing fractions tasks from a text by Empson and Levi (2011). The coach has purposefully sequenced story problems before problems without contexts so that students can consider the quantities in contexts that can be modeled. This lesson comes after students have had the chance to solve equal-sharing problems that have provided them with opportunities to partition wholes as well as name and write fractions (see also Lewis et al. 2015).

To examine and understand students' thinking, Soren and Tara walk around the classroom together and talk to students, who are at work on the problem set. As they examine students' work, they become curious about students' use of representations to model the problems. They stop for Soren to engage in a conversation with Leila, who was new to Hilltop at the beginning of the year. The teacher and coach are unsure of Leila's previous experience pertaining to learning fractions.

Soren: Leila, can you explain to us how you are thinking about this problem?

Leila: Well, I know I need to add two and six-eighths with one and five-eighths. I know that two plus one is three. [*Her voice trails off.*]

Soren pauses. From looking at Leila's work (see **fig. 2a**), he guesses that she might be stuck trying to add sixth-eighths and five-eighths. So, he asks her a question: "OK, so you want to add these numbers [*pointing to 2 6/8 and 1 5/8*]?"

Leila nods.

Soren: And so you added two and one, which is three. Do you have an idea of what you want to do next?

Leila: I don't know how to add these parts [*pointing at* 6/8 *and* 5/8 *in the equation*].

Soren: OK, can you tell me what you drew here? [*He points at the squares.*]

Leila: I wanted to draw the pizza, but I don't know how to show these [*pointing at the denominators*].

Soren: Do you remember what those are called?

Leila: Eights?

Soren: We call those eigh*th*-size pieces. [*He writes "eighth-size pieces" on her paper.*] Do you mind if I tell Ms. Tara what we just talked about? We'll be right back to help you think about the eighth-size pieces.

Leila: OK.

Tara and Soren step away from Leila so they can converse about Leila's thinking.

FIGURE 1

Fourth graders worked in pairs on a set of problems that gave them opportunities to add or subtract mixed numbers with like denominators.

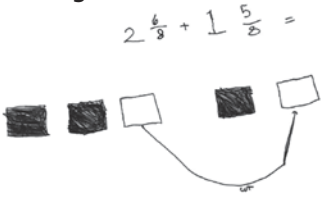
1. Ms. Nguyen's class ate 2 6/8 cheese pizzas and 1 5/8 pepperoni pizzas. How much pizza did they eat altogether?
2. In the refrigerator were 3 1/4 pizzas. The children ate 1 3/4 pizza for lunch. How much pizza was left?
3. 1 1/8 + 2 6/8 =
4. 2 6/10 – 1 1/10 =
5. 2 2/4 + 2 3/4 =
6. 2 3/6 – 5/6 =

FIGURE 2

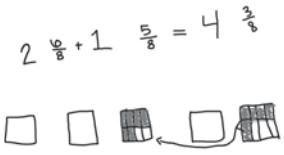
Students were asked to make a drawing of their strategy to solve a fractions question.

Mrs. Nguyen's class ate 2 6/8 cheese pizzas and 1 5/8 pepperoni pizzas. How much pizza did they eat altogether.

(a) Looking at Leila's work, her teacher suspected that she might be stuck trying to add sixth-eighths and five-eighths.



(b) Muna was able to draw her process, so the math coach suggested to the teachers that Leila and Muna talk about how to make eighth-size pieces.



Tara: That was a nice conversation. You did a good job asking Leila questions to uncover her thinking. What do you think she understands about fractions?

Soren: She's trying to use drawings but doesn't know how to show eighth-size pieces. She told me, "I don't know what to do with six-eighths and five-eighths."

Tara: What do you think we should do to support her?

Soren: I was wondering if we should help her partition the wholes into eighth-size pieces. Or should we have her work with Muna? She was able to partition into eighth-size pieces [*see* **fig. 2b**].

Tara: I saw Muna's work and agree that we could suggest Leila and Muna talk about how to make eighth-size pieces. What will you do to support the conversation between Muna and Leila?

Soren: While Leila is listening, I'll ask Muna to show us again how she cut some of the pizzas into eighths. Then I'll have her show how she added all of the eighth-size pieces together.

Tara: That sounds good, and we can ask Leila questions to see what she understood about how Muna partitioned and see if she could also try partitioning into eighths. We might also want to consider having a similar conversation about partitioning with the whole class as well. OK, let's go talk with Muna and Leila.

In this exchange, Tara starts her coaching conversation by asking Soren what he thinks Leila understands about combining mixed fractions. By doing this, the coach invites the teacher first to focus on student thinking and second to consider instructional implications on the basis of their shared assessment of what students know and can do. In this brief conversation, the coach and the teacher work on multiple aspects of eliciting and using evidence of student thinking (NCTM 2014), including when and how to support an individual student; when to orient students to others' thinking; how to support students to use a visual representation; and how to consider student thinking, of individuals as well as across the classroom, to make decisions about where to steer the concluding whole-class discussion. Together, they decide to focus the whole-class discussion on the first question so they have time to discuss some of the important mathematical ideas needed to engage in adding fractions that emerged when they watched Leila, Muna, and other students.

Goals for teacher learning: Supporting students' transition

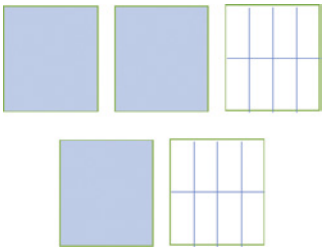
To support teachers in eliciting and making use of students' thinking, Tara aims to add to teachers' knowledge of mathematics learning trajectories, which are "how students make transitions from their prior knowledge to more sophisticated understandings" (NCTM 2014, p. 13). A learning trajectory for adding and subtracting mixed fractions with like denominators includes students first acting on fraction quantities with concrete representations, such as drawings showing same-size units, to understand the need for common denominators (see **fig. 3a**). Later, students can abstract toward more sophisticated ways of reasoning, such as using a counting strategy in which the student starts with one quantity and adds the second

FIGURE 3

Here is a potential model of how students' solution strategies may develop toward more sophisticated and efficient ways of reasoning over time.

Mrs. Moffatt's class ate 2 6/8 cheese pizzas and 1 5/8 pepperoni pizzas. How much pizza did they eat altogether?

(a) Modeling strategy



(b) Counting strategy

"I started at 2 6/8 and then added 1 to get 3 6/8. And then I added the 5/8: 3 7/8, 4, 4 1/8, 4 2/8, 4 3/8."

(c) Strategy that uses properties of operations

$$\begin{aligned} 2\ 6/8 + 1\ 5/8 &= 2 + 1 + 6/8 + 5/8 \\ &= (2 + 1) + (6/8 + 5/8) \\ &= 3 + 11/8 \\ &= 3 + 1 + 3/8 \\ &= 4 + 3/8 \\ &= 4\ 3/8 \end{aligned}$$

Developing along a trajectory

Teachers help students work toward solving this problem by understanding that fractions are a sum of unit fractions (e.g., 5/3 = 1/3 + 1/3 +1/3 +1/3 +1/3). With this insight, students decompose and compose fractions with the same denominator as a way to flexibly add and subtract fractions. By the end of fourth grade, according to the Common Core State Standards for Mathematics (CCSSI 2010) students should be able to compute sums of whole numbers and fractions through representing the whole number as an equivalent fraction with the same denominator as the fraction (e.g., 4 2/3 = 12/3 + 2/3 = 14/3). Students should also be fluent in converting mixed numbers into a fraction greater than one (e.g., 3 1/5 = 16/5) and vice versa. Repeated reasoning with these ideas throughout fourth grade with problems that gain in complexity leads to fluency and flexibility in solving these problem types.

quantity a little bit at a time until both quantities are added (e.g., 2 6/8 + 1 + 1/8 + 1/8 + 1/8 + 1/8 + 1/8). This could be followed by a more efficient strategy that involves using properties of operations, such as the associative property of addition (see **fig. 3b**), and creating algorithms for adding and subtracting fractions. With these learning goals in mind for students and teachers, we examine a second dialogue in which the coach enters the coaching conversations through first focusing on students' thinking.

Teachers' collaborative coaching conversation

Sue and Daniella are Soren's fourth-grade colleagues. The same day that the first classroom conversation takes place, Sue and Daniella have given their fourth graders the same set of problems (see **fig. 1**). Tara has also visited Sue's and Daniella's classrooms that day. As she circulates around the classrooms, Tara notices that teachers are supporting students in different ways to use mathematical representations. Sue is encouraging all her students to use a drawing for the first problem and then giving them the option to use a drawing, if needed, for the remaining problems. By contrast, in Daniella's class, Tara does not see many students using drawings. Tara aims to (1) bring to the surface differences she has noticed in the instruction across classrooms and (2) discuss students' use of representations in their transition toward more sophisticated ways of reasoning. Tara aims to support teachers in giving students opportunities to experiment with representing problem situations in multiple ways (SMP 4, modeling with mathematics).

An extended conversation takes place between the coach and the fourth-grade teachers, who have brought in samples of their students' work from the lesson to examine together.

Tara: What do your students understand about adding and subtracting mixed numbers? What are they confused about?

Daniella: I noticed that most of my students understand that a mixed number represents a quantity rather than treating the numerator and denominator as separate numbers.

Tara: How do you know that?

Daniella: Because they're able to show a mixed number in a drawing. Like if they had four and two-thirds, they would draw five squares. They

would shade in four of the squares to represent four wholes and partition the fifth square into thirds, shading two of those parts.

Soren: Tara and I noticed today that some students were having problems drawing eighths.

Sue: I was curious about that too, so I asked all of my students to represent the first problem with a drawing [*see* **fig. 1**]. I knew that not all of my students needed to use a representation to solve the problem, but I wanted to see what everyone would draw. For the rest of the problems, I told them to make a drawing only if they wanted to.

Tara: So, that leads me to wonder: When do students need to use a visual representation, and when do they not need to do so?

Daniella: Don't they need to be able to draw the quantities before they can abstract to visualizing the quantities without relying on a concrete representation?

Sue: I think so. We found out last year that it was important to give our students opportunities to draw the quantities. Later in the unit, we started encouraging our students to move away from drawings as they developed other strategies for solving the problems.

The coach begins this conversation by focusing teachers' attention on student thinking, asking what they notice about their students' understanding. Eliciting teachers' observations enables the group to consider together how to support students' reasoning with fractions and what to do next with instruction. By asking this question, the coach helps teachers develop the habit of focusing on student thinking. Further, she helps teachers consider the trajectory along which students' learning progresses when she asks, "When do students need to use a visual representation and when do they not need to do so?" Next, the coach supports teachers to think about how they will adjust their instruction in response to what they have learned about students' current understandings.

Tara: As we move forward in the unit, we need to consider this: How do we help students move from directly modeling the situation to using more efficient strategies?

Soren: I was thinking that I'd like to begin my lesson tomorrow by asking students to add three and two-eighths plus two and seven-eighths because I'm curious how students will deal with the fractional parts and having to create another whole out of combining two-eighths and seven-eighths.

Tara: Let's anticipate what we would want students to do or hear students say.

Sue: If we posed three and two-eighths plus two and seven-eighths to our students, I'd anticipate we would see them taking one-eighth from the two-eighths to combine with seven-eighths to make a whole.

Soren: And we might also see them combine the wholes first, then the fractional parts.

Daniella: I think I would like to give them whiteboards or a piece of paper to solve the computation. It would be interesting to see who relies on a drawing to add the quantities.

Tara: So when the lesson is enacted, we would want to think about which strategy we would like to have shared with the whole class first and why. And which strategy would we have shared next, and what connections would we want students to make between them?

Soren: Linking the drawings students make with strategies that are more abstract could be the goal of the discussion, because we want to support the students who are ready to move away from drawings.

Tara: Daniella and Sue, do you think beginning your lesson in a similar way would help your students?

[*Both teachers nod in agreement.*]

Sue: Let's also all give a common exit ticket across the fourth grade to see if our students use drawings or try other strategies to solve addition of mixed numbers.

Tara: We can bring the student work to our next collaborative meeting.

In the previous conversation, the coach presses teachers to consider how they would support students' transition to more abstract ways of reasoning by intentionally selecting student work to share during a whole-class discussion and making connections between the strategies and the representations (Kazemi and Hintz 2014; Stein and Smith 2011). She accomplishes this by asking the teachers

about which strategy they will share first and why. Further, the coach supports the teachers' ideas of designing a fractions task and helps them to consider a mathematical goal they will work toward in their whole-class discussion. She appreciates their desire to establish a common exit ticket so they can examine the students' work during the following week's collaborative meeting to gauge the progress that students are making along a trajectory for making sense of fractions.

Influencing instruction, improving learning

In our own experiences as coaches, we get excited when teachers come up to us in the hallway to relay their students' mathematical thinking. When this happens, we know that teachers have sharpened their attention to student thinking and have begun to establish classroom communities where they are eliciting and highlighting student thinking so that students can make sense of mathematics together. By beginning our coaching conversations with a focus on student thinking and by asking questions, we can make a positive impact on instruction to influence student learning. When coaching conversations focus not solely on changes in the teacher's practice, but instead on changes in instruction in response to student thinking, the motivation to change practice can come from a desire to do what is best for students rather than to merely conform to a prescribed course of best practices suggested by the coach. Teachers who are striving to meet the needs of their students can feel empowered to make significant changes in their teaching.

Common Core Connections

4.NF.B.3A 4.NF.B.3.C
4.NF.B.3B 4.NF.B.3.C

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