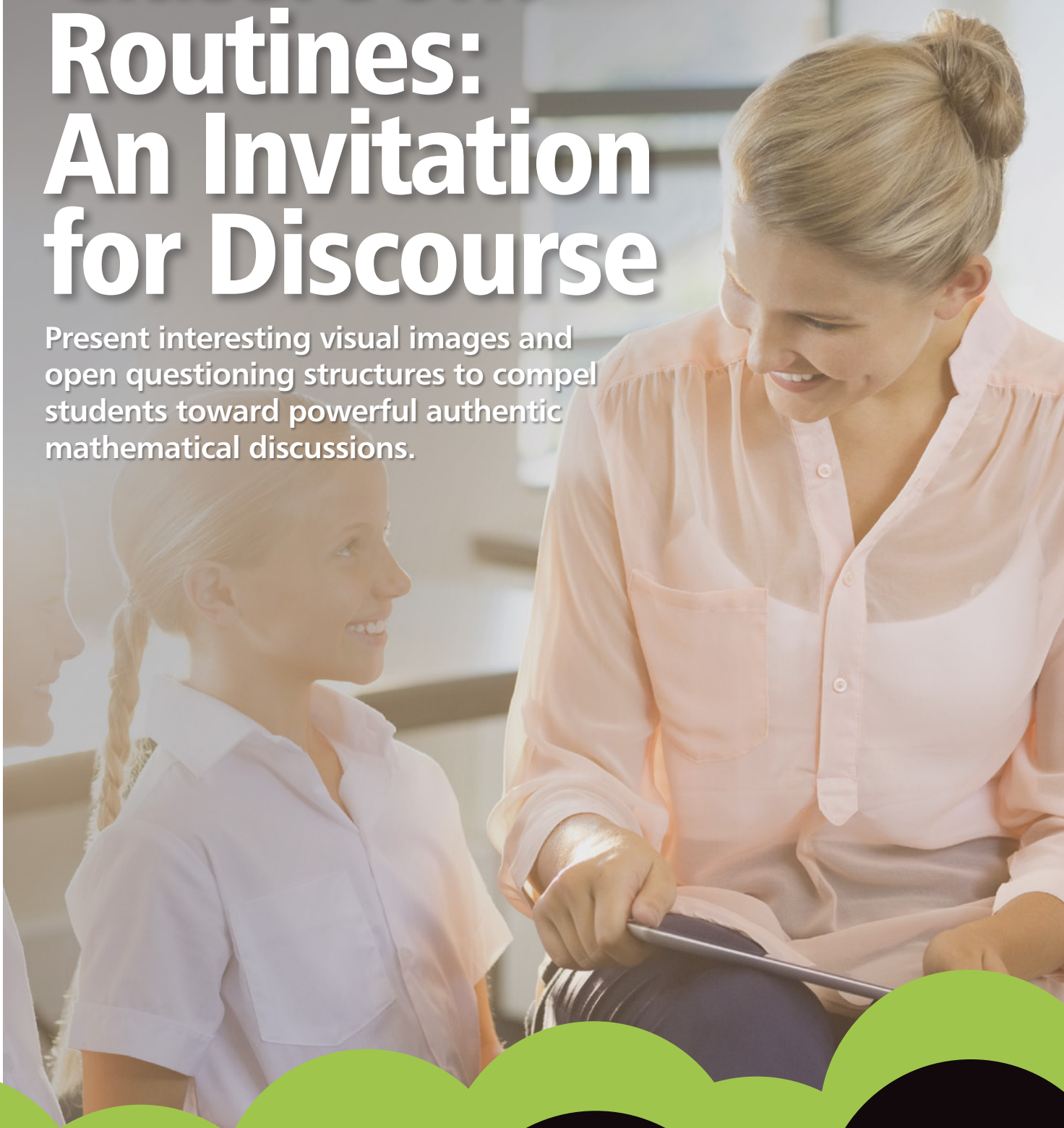


Classroom Routines: An Invitation for Discourse

Present interesting visual images and open questioning structures to compel students toward powerful authentic mathematical discussions.



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Intentional Mathematical Discourse

Christine Newell and Chase Orton

Picture this: An interesting image is presented to a group of students. They begin buzzing excitedly about what they see as the teacher poses a question inviting them to think mathematically about the image. With a visual prompt and a purposeful question, students engage in authentic discourse that moves the learning of the whole class forward while the teacher records thinking and supports students as they make connections between ideas.

Mathematical routines can provide opportunities for students to engage in this kind of meaningful discourse. A mathematical routine is a short activity with a specific structure that allows students to engage in sense making through discourse, reasoning, and justification. The Twitter community of math educators (found through the hashtags #iteachmath and #MTBoS) regularly share and collaborate about powerful mathematical routines. Many of these routines revolve around visual images and the rich discourse that results when students engage with them.

Why discourse matters

Principles to Actions: Ensuring Mathematical Success for All calls for teachers to foster a classroom environment built on meaningful mathematical discourse (NCTM 2014, p. 29). Mathematical discourse is “the way students represent, think, talk, question, agree, and disagree in the classroom” (Stein 2007). Whether the discourse is between teacher and student or between student and peers, talk is an essential component of developing student understanding. According to Carpenter, Franke, and Levi,

[Students] who learn to articulate and justify their own mathematical ideas, reason through their own and others’ mathematical explanations, and provide a rationale for their answers develop a deep understanding that is critical to their future success in mathematics and related fields. (2003, p. 6)

When students are challenged to think and reason about mathematics and communicate the process or results of their thinking with others, they learn to be clear and convincing in their verbal and written explanations. Additionally, listening to others explain gives students opportunities to develop their own understanding (NCTM 2009).

This article highlights four routines that use visual images and open questions to invite students to discuss, justify, make arguments, and question their own thinking as well as their

classmates’ thinking. We will share specific classroom examples from each routine that highlight the meaningful discourse they can inspire. Teacher prompts and questions that are essential to each routine are in a bold typeface.

Routine 1: Number-talk images

First-grade students are settled on the carpet, ready for a number talk about the picture of cupcakes (see **fig. 1**).

Teacher: How many cupcakes do you see in this box? How do you see them? *[Students are given one full minute of wait time to think individually. They excitedly start showing a thumbs up when they have an answer and a strategy they want to share for counting the cupcakes.]*

Teacher: Let’s say the number together. How many cupcakes?

Students: Twenty!

Teacher: Turn and talk to your partner about how you knew there were twenty. *[Students turn knee-to-knee with their partner and share strategies. The teacher calls students back together and asks if anyone would like to share with the class.]*

Student 1: I counted them.

Teacher: What did it sound like when you counted them?

Student 1: I counted by ones, like this: one, two, three—all the way to twenty.

Teacher: Great! Did anyone count them a different way?

Student 2: I counted by fives.

Teacher: Where did you see fives?

[Student 2 comes up to the poster and shows where he sees groups of fives in the image.]

Teacher: Let’s count by fives together. Five, ten, fifteen, twenty! Thank you for sharing. I wonder if anyone knew there were twenty a different way?

Student 3: I saw four plus six plus five plus five, and that is two tens, so it made twenty.

Teacher: Hmm, she saw four plus six plus five plus five. Where do you think she saw the four and six and five and five? Turn and talk to your partner.

FIGURE 1

This is one of hundreds of images curated at the Number Talk Images website, ntimages.weebly.com. Elham Kazemi shared this picture.



Number talks with images can spur a meaningful math discussion in classrooms at any grade level. To facilitate a number-talk image discussion, teachers intentionally select an image or images as an engaging focal point for students and ask them to share how many items they see and how they see them. As students share, the teacher records ideas publicly by annotating the image (see **fig. 2**) or capturing ideas in words or equations.

Pierre Tranchemontagne (@Pierre_Tranche), a Canadian math educator, began the Number Talks Images website as a response to a request from teachers for a one-stop spot for the interesting images that people were sharing on Twitter. According to Tranchemontagne, the goal of using visual images to launch a number talk is to provide an entry point for all students in a way that word problems or numbers alone do not. The engaging images promote multiple ways of seeing a quantity by counting or subitizing, and the open questions encourage students to share their strategies (Pierre Tranchemontagne, e-mail to author, October 21, 2017). The collection of number-talk images is always being added to and can be found at <http://ntimages.weebly.com>.

Routine 2: Which one does not belong?

During a Which One Doesn't Belong? (WODB) routine, students are encouraged to analyze and compare visual images while defending their thinking in conversation. WODB begins by showing students four images of shapes, graphs, numbers, expressions, or equations, followed by asking them, "Which one doesn't belong? Why?"

After a period of silent think time, students look for similarities and differences in observable attributes as well as relationships and patterns while considering the language they will use to communicate their rationale. Students first think independently and then are invited to share with a partner, group, or the whole class. Teachers can record students' reasoning next to or below the images as students explain (see **fig. 3**).

Teacher: [Displaying **fig. 4**] Which of these four shapes does not belong, and how do you know?

[Students think silently and put a thumb up]

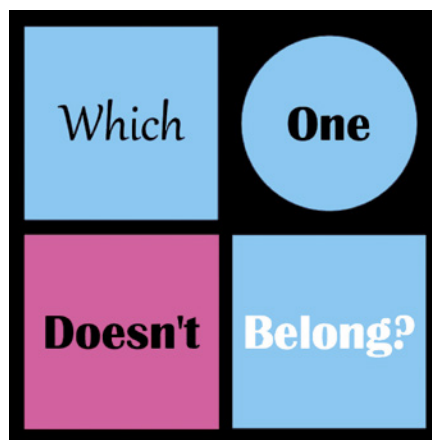
FIGURE 2

As students share their thinking, the teacher annotates to help make strategies visible to others.



FIGURE 3

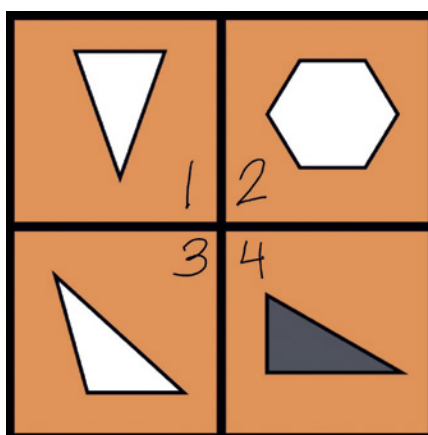
Which One Doesn't Belong? focuses on justification and multiple approaches.



SOURCE: WODB.CA (2)

FIGURE 4

Each Which One Doesn't Belong? image has four options for students to consider. The teacher can label each one to support student discourse.



when they have an answer and a justification.]

Teacher: Thumbs up if you chose one that doesn't belong. **Share with your partner, "I think shape _____ doesn't belong because..."**

[A conversation takes place among three students.]

Student 1: I think the one that looks like a stop sign doesn't belong because it's like a hexagon and the other ones have triangles.

Student 2: I think it's shape 2 because it's a stop sign and not a triangle.

Student 3: I agree because it doesn't have three sides on it like the other ones.

Teacher: [Bringing the group back together] **Who would like to share their thinking or their partner's thinking? Which one doesn't belong and why?**

Student 4: Number 4 doesn't belong because it's a different color—it's all dark.

Teacher: So, what can we say about the other three?

Student 5: Well, shapes 1, 2, and 3 all belong because they are white.

Teacher: I'm going to give you a challenge: **Can you think of a reason why shape 1 doesn't belong?** Turn and talk.

FIGURE 5

A first grader explains to his partner what the "upside down triangle" should look like.



[A conversation takes place between two students and the teacher.]

Student 1: The first one doesn't belong because it's upside down!

Teacher: What do you mean by "upside down?" Can you show me?

Student 2: It should look like this [demonstrating with his hands; see fig. 5].

Christopher Danielson (@Trianglemancsd), author of *Which One Doesn't Belong? A Shapes Book*, explains in his book that—

each set [of images] is designed in such a way that any of the four can be seen as different from the other three. This small adjustment takes a simple activity about noticing sameness and difference and transforms it into a challenging task that supports rich conversations requiring precise language use and sophisticated argumentation. (Danielson 2016, *Teacher's Guide*, p. 1)

When facilitating a WODB routine, "It is essential to make sure that everyone understands that any one of the four objects can be correct and that there are multiple possible reasons for each one not to belong" (Danielson, e-mail message to author, November 16, 2017).



WODB routines challenge students to explain and justify their reasoning.

By opening up the answer possibilities to anything that a student can logically justify, WODB invites students to make an attempt without the fear of being wrong. Teachers have reflected that when students trust that their ideas will be valued, they will share these ideas openly and with pride. If a student's claim is incorrect, teachers and other students are invited to challenge it respectfully (Danielson 2016). WODB owes its creation to several people. Danielson acknowledges Megan Franke (@meganlfranke) and Terry Wyberg (@TerryWyberg) for their inspiration. Mary Bourassa (@MaryBourassa) also curates a collection of WODB images at wodb.ca, and other images can be found on Twitter at #wodb.

Routine 3: Estimation 180

At Estimation 180 (see **fig. 6**), visual estimation challenges inspire students to engage in mathematical conversations and to support their mathematical claims with evidence and reasoning. Math educator Andrew Stadel (@mr_stadel) developed Estimation 180 to provide “students with opportunities to strengthen their number sense and mathematical thinking through the use of engaging visuals and rich

FIGURE 6

Estimation 180 invites students to estimate and reason.



SOURCE: ESTIMATION180.COM (2)

FIGURE 7

Students are shown these images of a bowl of coins and asked to share estimates and reasoning.



discourse” (Andrew Stadel, e-mail to author, October 21, 2017).

During an Estimation 180 activity, students are presented with an image and asked to make a series of estimates. Students start by making estimates that they know are “too low” and “too high” before choosing a “just right” estimate. They then share their reasoning before the actual answer is revealed. Students are encouraged to use their own experiences, the visual provided, and previous estimation challenges to strengthen their estimate and analyze their accuracy once the answer is revealed. Here is a snapshot of an Estimation 180 routine in a fifth-grade class:

Teacher: Look at this bowl of coins [see **fig. 7**]. You can see it from a few different angles, which will help you get a better idea

of what's in there. **What do you notice about this bowl of coins?**

Student 1: I notice there are different types of coins.

Student 2: I notice there are layers of coins.

Teacher: What do you think is the value of all the coins in the bowl? Write down an estimate that you know is too low. In other words, you're confident that there is more than that amount in the bowl. Turn and talk to a partner about your "too low" estimate.

[Students discuss in pairs and as a group.]

Student 3: I think seventy-five dollars is too low because I can see a lot of quarters, and it only

takes four quarters to make one dollar.

Teacher: So, the type of coins you see is having an impact on your estimate?

Student 3: Yeah, different types of coins would change the value completely.

Teacher: *[Students have written down and shared a "too-high" estimate.]* Now I want you to write down your actual estimate. **What's your estimate and your reasoning?**

Student 4: My estimate is 250 dollars because I followed my gut. I'm pretty sure that 200 dollars is too low and 300 dollars is too high.

Estimation levels the math playing field for all learners because it allows students to rely on intuition and reasoning skills in addition to making informal calculations. Starting with arguing for answers that are intentionally imprecise helps them hone their estimates and see themselves as owners of mathematical ideas and understandings (Bay-Williams et al. 2013). Some Estimation 180 activities can serve as entry points into longer, more formal math lessons that can target specific grade-level content. Stadel has created 300 estimation challenges (see estimation180.com) that can be used with students in any grade level, and more can be found on Twitter at #estimation180.

Routine 4: Would you rather?

Would You Rather? math (see **fig. 8**) is a routine that math educator John Stevens (@JStevens009) uses to inspire students to "construct viable arguments and critique the reasoning of others" (CCSSI 2010, p. 6). During a Would You Rather? routine, students are presented with an image or statement that describes a dilemma (see **fig. 9**), and they are required to "choose a path and justify it" (wouldyourathermath.com). After individual think time, students are invited to share their reasoning with one another and consider the arguments of classmates who disagree with them. In addition to verbal discourse, routines like Would You Rather? provide opportunities to ask students to justify their reasoning in writing as well. Consider reflections from a third grader (see **fig. 10**) and a fourth grader (see **fig. 11**) on a Would Your Rather? task.

According to Stevens, Would You Rather? prompts are designed to get students to debate

FIGURE 8

Would You Rather? Math invites students to choose a path and justify it.

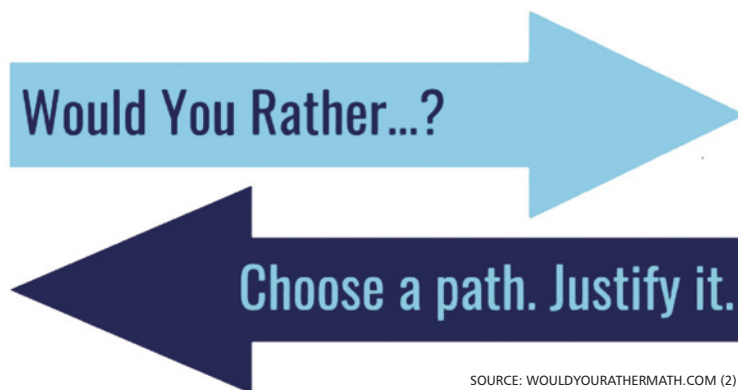


FIGURE 9

Students were presented with the question, "Would you rather have 38 baskets of 12 apples or 40 baskets of 10 apples?" After a class discussion, they were asked to write about their argument.



and argue using mathematics as the foundation for conversation. With valid mathematical or contextual reasons to justify both sides of the dilemma, but no “right” answer, student discourse focuses on argumentation, “a process of dynamic social discourse for discovering new mathematical ideas and convincing others a claim is true” (Rumsey and Langrall 2016). “With concepts that are curiosity-provoking and/or relatable to students’ experiences, there are plenty of ways to get to the ‘right answer’ as long as the reasoning is justified” (John Stevens, conversation with author, October 20, 2017). A collection of these challenges can be found at wouldyourathermath.com, and more can be found on Twitter at #wyrmath.

Other routines that invite discourse

In addition to the four routines described, the community of math educators on Twitter regularly shares other ideas, routines, and lesson structures that inspire student discourse:

- Notice and Wonder (#noticewonder) supports problem solving by slowing down the process and encouraging students to make sense of contexts and relationships before rushing to compute with numbers and keywords (Annie Fetter, @MFAnnie).
- Splat! asks students to use algebraic reasoning and conversation to determine the value of dots hidden under a “splat.” Steve Wyborney (@SteveWyborney) shares whole-number and fraction challenges on his website, stevewyborney.com.
- Same or Different? (#samediffmath) is a routine that provides students with opportunities to construct arguments when comparing objects, such as numbers or shapes. Brian Bushart (@bstockus) curates a community website of images at samedifferentimages.wordpress.com.
- Presenting problems as a three-part story, 3-Act Tasks start with an image or video that inspires students to ask and answer questions and justify their thinking. Graham Fletcher (@gfletchy) and Dan Meyer (@ddmeyer) share 3-act lessons on their websites, gfletchy.com and blog.mrmeyer.com.

FIGURE 10

A third grader reflected on the Would You Rather? question from figure 9.

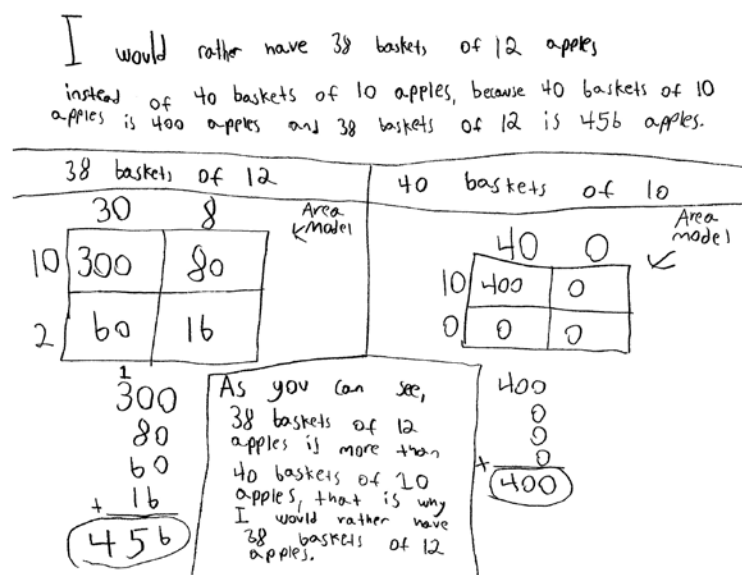
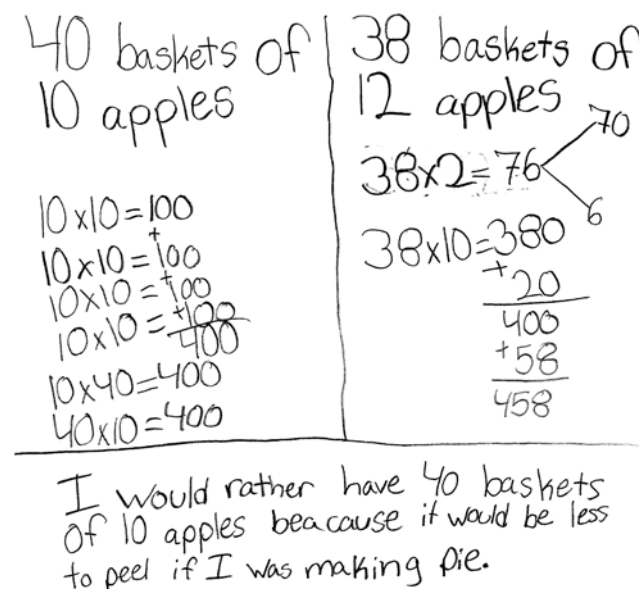


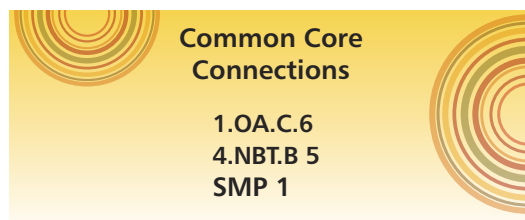
FIGURE 11

A fourth grader reflected on the Would You Rather? question from figure 9.



Conclusion

Throughout this article, we have discussed four routines that invite students to participate in discourse that advances the learning of the whole class as a mathematical community (NCTM 2014, p. 29). Each routine launches with an interesting visual image to motivate students to share their own thinking and consider the approaches of their classmates. By facilitating each routine with open questions, and by both honoring and building on student thinking, teachers can support students in seeing mathematical discourse as a tool to support understanding. In each classroom example shared, we have highlighted important questions that teachers can ask during each routine to invite students to the conversation. We hope that you will use these as inspiration and support to try them in your own classroom as you work toward the worthwhile goal of engaging all students in meaningful mathematical discourse.



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