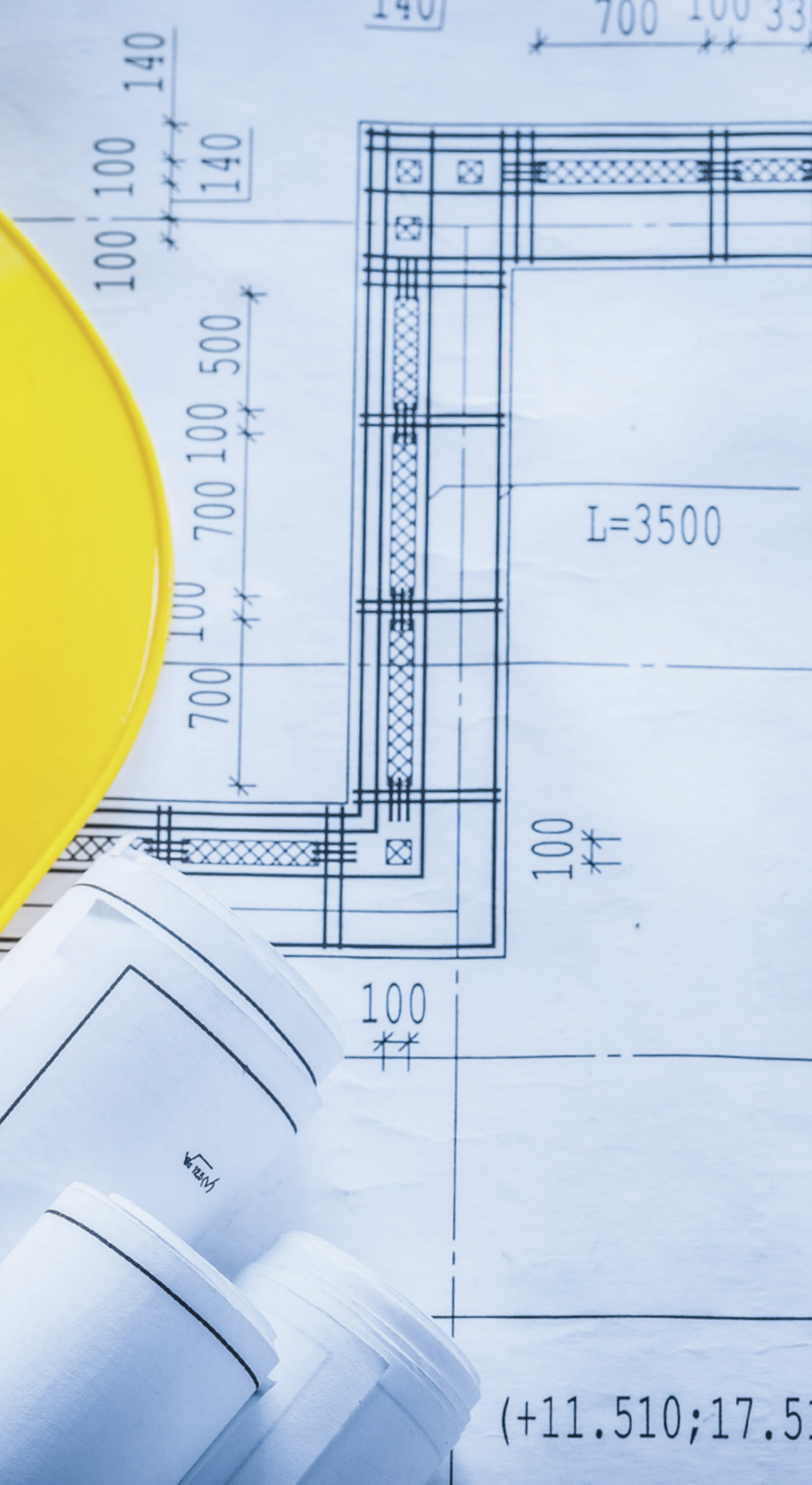




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BUILDING THE Optimal Learning Environment FOR MATHEMATICS

Teachers can use
Universal Design
for Learning
(UDL) in their
math classroom
to anticipate
potential
barriers, know
which tools will
engage students,
and provide
safe spaces for
learning.

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Remarkable differences exist in the ways in which students can be engaged or motivated to learn. According to Hilyard (2013), students who participate in class trust and have a connection with their teacher. Trust can be built over time between a mathematics educator and a student through respect, a safe environment for students, belief in a student's ability, care for students, and the competence of the instructor (Hilyard 2013). One of the most important roles of a teacher is to create a safe space for learners, consistent with the Center for Applied Special Technology (CAST) Universal Design for Learning guidelines (CAST 2011). Universal Design for Learning (UDL) is a framework for teaching and learning, based on neuroscience research, that guides teachers to eliminate barriers and allows the content and curriculum to be accessible to all students (Rose et al. 2006; CAST 2017). Using the UDL framework as they plan, teachers provide students with options to stimulate and maintain their interest to learn (multiple means of engagement), present information so that students can perceive and comprehend information (multiple means of representation), and offer strategies to students to organize information and express what they know and understand (multiple means of action and expression) (CAST 2011; National Center on Universal Design for Learning [NCUDL] 2014; Novak 2016). As teachers use the UDL guidelines, they anticipate having diverse individuals in their classrooms, and they plan their lessons and assignments with that diversity in mind. UDL, like differentiated instruction (DI), is a way to provide students with options to access challenging tasks

within the curriculum. Both UDL and DI require teachers to adjust the presentation of information to learners, as well as to vary the requirements for students' responses. However, UDL focuses on eliminating barriers through initial designs that consider the needs of diverse students, rather than overcoming those barriers later through individual adaptation. Using the UDL guidelines, teachers proactively create learning environments and pedagogy accessible to all students and encourage students to become more autonomous learners by embedding choices and removing learning barriers in their instruction (Novak 2016; Rose et al. 2006). Within UDL, teaching and learning are dynamic and flexible, and the curriculum is viewed as needing to be adjusted to fit students' needs, rather than adjusting students' needs to fit an inflexible curriculum.

Consider the following vignette of a first-year algebra classroom. How can Mr. Jones adjust his current classroom environment to make Adam feel safe to take risks and make mistakes as he learns mathematics? How can Mr. Jones structure his classroom

Setting up a classroom environment conducive to promoting discourse can make the difference in validating essential problem-solving skills.

to promote more mathematical discussion and meaningfully engage Adam using the UDL guidelines?

Mr. Jones models how to solve an equation and determine how many solutions it has; then he asks students to try a problem on their own. As students work, the math coach observes one student, Adam, as he expertly performs inverse operations, isolates the variable, and comes to a correct conclusion regarding the number of solutions. A few moments later, Mr. Jones brings the class back together to report on what students have discovered. He chooses Jenna, an eager volunteer, to describe the steps taken to solve the problem. As Jenna explains her work, the teacher writes the steps on the board for students to reference. Each step is logical and accurate, and Jenna reaches the conclusion that the equation has no solutions. The coach can see Adam stare at the white board, puzzled that his work is not identical to the work on the board. Certain that his procedure is incorrect, Adam erases everything that he had completed and copies what is on the board as the class transitions to the next activity. Although Adam excelled in completing the task at hand, he thought that his work was invalid because it was not exactly what the teacher wrote on the board.

Later, the math coach speaks to Mr. Jones about

setting up a classroom environment conducive to promoting discourse about multiple ways to solve a problem. For students like Adam, this can make the difference in validating essential problem-solving skills even though they are not identical to the procedure displayed on the board.

LAY OUT THE BLUEPRINT

Mathematics teachers must know the content and standards, but they must also know their students and what motivates them (Hilyard 2013). Teachers of mathematics not only develop conceptual understanding and procedural fluency in their students but also support students in believing that they are "legitimate and powerful doers of mathematics" (Aguirre et al. 2013, p. 14). Students develop a sense of agency in mathematics when they become active participants in their learning and persevere to solve problems (Aguirre et al. 2013). Teachers also play a pivotal role in facilitating coping skills and appropriately handling math phobias and judgments of natural aptitude (e.g., *How can I improve on the areas I am struggling in?* rather than *I am not good at math*) (CAST 2011).

When math teachers create safe learning spaces for math and minimize threats and distractions, students like Adam can actively engage in learning mathematics (CAST 2011). In this article, we explain steps that beginning teachers can take to build a safe space for learning math, anticipate potential barriers that might hinder them in the first few years of teaching, and suggest tools we have used in our classroom practice to deconstruct barriers and successfully engage students.

PREPARE THE SITE; POUR THE FOUNDATION

By applying a UDL lens to his algebra class, Mr. Jones can offer students multiple means of engagement, motivating them and providing a safe and positive space to learn (CAST 2011; NCUDL 2014; Novak 2016). A positive learning environment is one that reduces threats and negative distractions, even subtle ones, so that students can access the information. In a middle or high school mathematics classroom, some threats and distractions could include—

- students' focus on meeting their basic (physical, social/emotional) needs;
- students avoiding negative or anxiety-causing experiences, such as believing they are not a "math person" or earning poor grades (Boaler 2015); and
- students lacking the necessary scaffolds and supports—especially English learners and students with disabilities—to access the vocabulary and content.

Adding strategies to your toolkit to create safe spaces for students will enable you to engage them in mathematical content knowledge and understanding throughout the year. Students can concentrate on learning mathematics content when they feel safe and confident that they will have a positive experience, as opposed to trying to avoid a negative experience, each time they enter your math classroom.

COMPLETE THE ROUGH FRAME; PROTECT AGAINST OUTSIDE THREATS

Remember to anticipate barriers that may threaten your ability to engage students. When Mr. Jones first started using the UDL framework and multiple means of engagement, he thought, *What if students don't understand the content? And what if students get too loud?* Balancing classroom management and mathematics content delivery while engaging students may be difficult. You may want to focus solely on transmitting content knowledge to students because maintaining control of the environment is easier if students are expected to copy notes quietly as you write on the board. Also, engaging students in mathematical discourse might result in a loss of control of the content and learning objectives. Mr. Jones also thought, *I don't have the confidence to cover the material in a way different from traditional math instruction* (e.g., go over last night's homework, introduce the new topic on the board, have students complete a few problems in class, assign more homework). *I just don't feel comfortable moving away from the way I was taught.* Many teachers teach mathematics as they were taught—through procedures. Beginning math teachers can “break the chain of how they were taught” and engage students in mathematics instruction (Faulkner 2009, p. 25). All these barriers can be acknowledged and addressed from the first day of a teaching career if you focus on building a safe and engaging environment for your students.

INSTALL INSULATION

Mathematics teachers can create an accepting, comfortable, and supportive classroom climate that minimizes threats by using the UDL guidelines on engagement (CAST 2011). Below, we suggest three incremental steps that teachers can take to foster a safe learning environment and engage students in mathematics.

Step 1: Communicate clear expectations for learning and behavior goals

To balance effective classroom management with rigorous mathematics content delivery, be explicit about expectations. After explaining a task expectation to students, have one or two students revoice it to the class, connecting it to the goal of the les-

son. This routine can help reinforce what students should be able to do by the end of class, and it can add a layer of accountability to the task. It can also reveal any gaps in vocabulary that students may have with the content-specific language of the task. Mr. Jones and the math coach designed a lesson in which students worked in groups to practice the content skill of the lesson. Students were told that each group would be responsible for presenting its work to the class for one randomly chosen problem. Mr. Jones showed students an exemplar, and students used this tool as a reference when they worked through a different example. The accountability of presenting a problem (with explicit expectations) facilitated more peer-to-peer talk and increased task persistence. The presentations also allowed students more practice in using math-specific vocabulary to explain their thinking and problem solving.

Mr. Jones also provided language support to the many English learners (ELs) in his classroom so that they could reach the same standard during this lesson. Mr. Jones put structures in place to give students access to content-specific math vocabulary, without decreasing the expectation for rigor. To support ELs, Mr. Jones used an anchor chart with key math vocabulary (such as *inverse operation*, *variable*, *greater than*, *less than*, *equal to*) and posted sentence starters to support students' oral reasoning skills. Some of these sentence starters include, “I agree with _____ because . . .,” “I also noticed that . . .,” “Another way to solve it would be to . . .” Giving the entire class access to these tools allows all students to feel supported, without singling out anyone.

Within the UDL framework, learning expectations are reachable by every learner (NCUDL 2014). Teachers can ask themselves, *Are the expectations clear? Do learners know what success looks like?* (CAST 2016, p. 2). In a supportive and risk-taking environment, students are motivated to push themselves beyond baseline expectations (CAST 2016). Positive feedback from the teacher promotes expectations and beliefs that optimize motivation, and students feel safe to take risks (Novak 2016).

Step 2: Engage students through discussion

When new teachers want students to reason numerically and justify their strategies and thinking to their peers, they can use number talks (Humphreys and Parker 2015). To facilitate a number talk, teachers can anticipate student needs and add appropriate scaffolds. First, teachers should review classroom norms around number talks, including behavior expectations. Sentence starters (“I think . . . because . . .” or “I approached/solved the problem by . . .”) posted in the classroom are helpful tools for students

Table 1 Barriers to and Strategies for Constructing Safe Spaces for Learning Mathematics

Barriers to Engagement	Small Steps to Increase Student Engagement
Solely focusing on transmitting the content knowledge to students	Connecting with students and develop a trusting relationship by providing a safe environment
Struggling to maintain control of the classroom while students engage in mathematical discourse	Establishing classroom expectations and norms; providing explicit expectations for tasks and exemplars; modeling productive number talks
Creating one-size-fits-all lessons	Developing predictable routines with some flexibility in the level of the task and groupings
Trying to do it on your own	Collaborating with skilled specialists and colleagues; asking the math coach to plan a lesson with you

who are unsure of how to frame what they are trying to say and for providing models for how to disagree politely with a peer. Finally, modeling what a productive number talk looks like in the classroom can be helpful. Before students launched into their turn-and-talks, Mr. Jones and the math coach used the script in the **sidebar** to model the peer-to-peer talk that was expected in this classroom for a lesson pertaining to inequalities.

Creating strategic, yet flexible, groups with differentiated tasks can also help in supporting student engagement and confidence. On the basis of student needs and data from exit tickets, Mr. Jones and the math coach created three groups of students. Those who were not yet proficient received extra support from the paraprofessional and co-teacher in the classroom, while advanced students worked together to deepen their understanding of the concept. Students typically notice and appreciate that these experiences have been individualized on the basis of their needs, and they take advantage of such opportunities. However, groups are flexible and alternate with mixed-ability groups for other learning objectives.

Step 3: Be flexible with your lesson implementation

Teachers can increase the predictability of daily activities and transitions by using consistent structures and protocols that become familiar to students

over time. Creating class routines, such as a daily *Do Now* problem, adds routine and implicit start-of-class expectations for students. Similarly, increasing the routine of presenting work helps students become more comfortable communicating their ideas using content-specific language. Mr. Jones's students have become accustomed to his classroom routines. They appreciate the structure.

Although maintaining structure is a goal, teachers can be flexible and vary the pace of work by adjusting the task on the basis of students' needs. When planning, anticipate areas in which students may need reteaching. If students clearly do not understand, sustain a focus on one objective or big idea that they should leave your class with that day, and consider a small plan B. If a student needs more practice with the basic skills of a standard, perhaps require him or her to complete only a specific portion of the task. Conversely, give a student who is proficient in the standard an extension to add rigor to the task.

The math coach and Mr. Jones discussed the importance of being flexible with plans and capitalizing on teachable moments when students were not on track after completing a class *Do Now*. Later that day, after having students complete a *Do Now* and realizing that at least half the class was completely lost in formulating inequality equations from word problems, Mr. Jones did not move forward with the planned instruction but instead chose to conduct a twenty-minute intervention session. Mr. Jones worked quickly to create strategic groupings of students. Proficient students were paired with struggling students to work additional problems collaboratively while Mr. Jones himself worked with a group of students who had missed the previous class to catch them up. Mr. Jones added flexibility to his lesson structure: After looking at their work, he adjusted his instruction according to his assessment of students' needs.

Within the UDL framework, teachers acknowledge learner variability and offer more options and alternatives—varied pathways, tools, strategies, and scaffolds for reaching mastery (NCUDL 2014).

Peer-to-Peer Talk Script

During a turn-and-talk, students turn to their neighbor to clarify and share ideas (Kazemi and Hintz 2014). The teacher circulates and listens to this partner talk. Here is a sample script.

Student A: After reading the task, I think it is asking us to. . .

Student B: I (agree/disagree) with what you said. I think that we should _____ first.

Student A: I agree. Let's both work on that first and then compare our work. (Or: I think another strategy would be to. . .)

Student B: Let's compare our work to see if we agree or need to review what we did.

When teachers craft goals that promote clear, high expectations for all learners and use flexible methods and materials, they create safe spaces in which students are engaged in learning mathematics.

KEEP YOUR TOOLS SHARPENED

As you put the structures in place to foster a safe classroom environment, you are on your way to becoming a great teacher. However, keep your engagement toolkit from getting dusty by continually adding new strategies and ideas through collaboration with others. **Table 1** summarizes potential barriers that teachers may encounter in engaging students with math content, as well as the strategies we have implemented in our classrooms to deconstruct these barriers, using the UDL guidelines.

When teachers anticipate barriers to learning, such as distractions, they can build supports into each lesson to help students engage with math content. Returning to our initial scenario, Mr. Jones has now made incremental changes to his instruction to ensure that he highlights multiple ways to solve a problem so that students feel safe in sharing their problem-solving techniques. Mr. Jones now models new content, allows students to discuss their answer with peers, and solicits multiple solution pathways. Following a lesson, he collaborates with the algebra team, the special education co-teacher, and/or the math coach to provide additional accommodations and support in subsequent lessons. Incorporating these small steps in individual lessons will build an engaging and safe learning environment for your students.

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