Place Value as a Rate of Ten

Development of this multiplicative process involves unitizing within and across place.

James Brickwedde

Consider the number 777. How many tens are in the number 777? How many tens are in the tens place? These are two distinct questions, both of which speak to aspects of a robust understanding of place value. Being able to determine the value within location, as well as to unitize across place (Lamon 1996), is useful in operating on numbers. A fourth-grade content standard from the

Common Core State Standards for Mathematics (CCSSM) state the need to recognize that in a multidigit whole number, a digit in one place represents ten times what it represents in the place to its right (CCSSI 2010, 4.NBT.1). The standard expands at fifth grade to indicate that a digit represents one-tenth of what it represents in the place to its left (5.NBT.1). Using computational strategies based on place value and properties of operations is common in second- and third-grade standards; such strategies form the foundation on which fourth- and fifth-grade mathematics is built.

This article examines the importance of developing the notion of place value as a rate of ten. In exploring how to nurture this concept, we look at the role of the language of value, the problem types of multistep multiplication and addition along with measurement division, each with ten as an organizing unit, as well as strategically crafted number strings to foster thinking in scale. The work described here is drawn from instructional and research work in classrooms during the past six years.

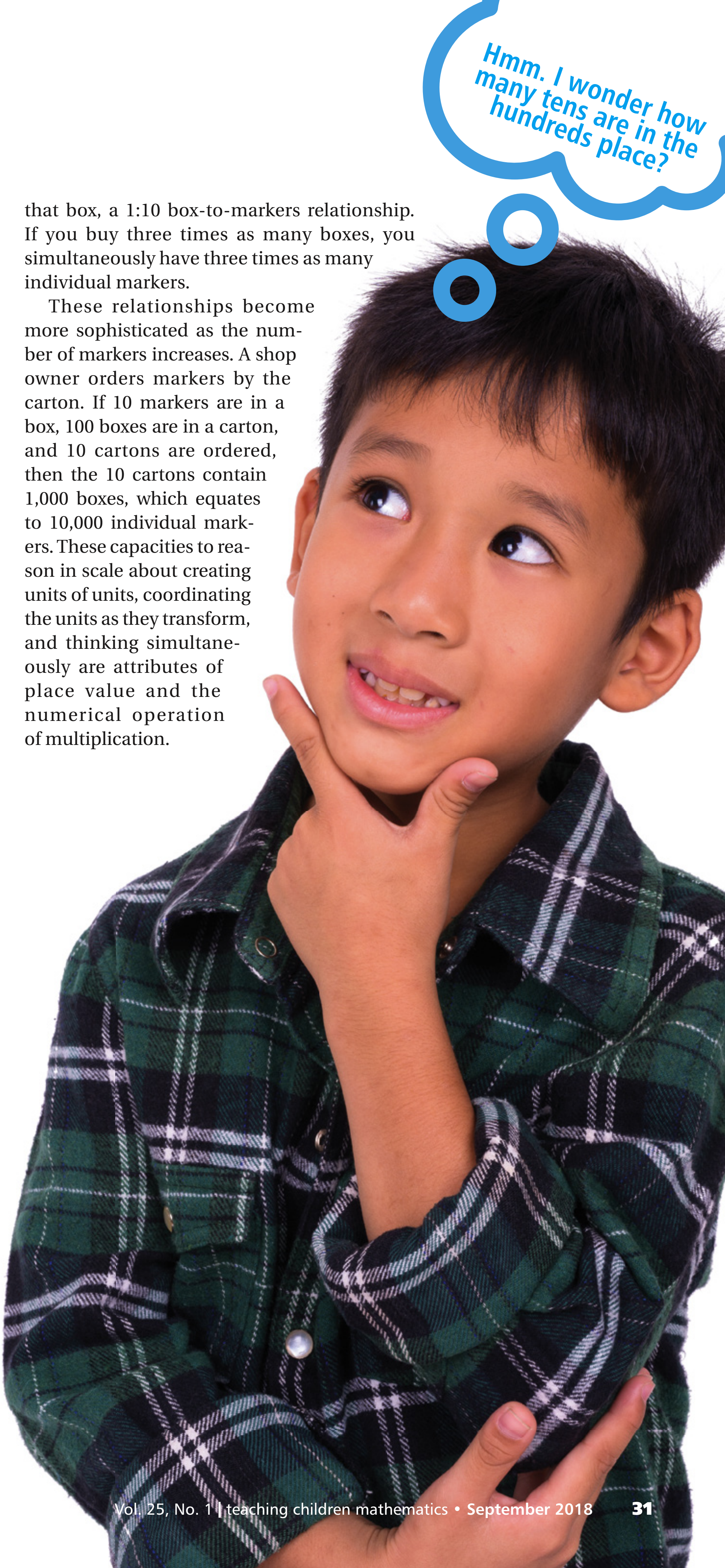
In 2016 the population of the United States was estimated to be 324.78 million people. The number 324 million is clear. But how many people (ones) is 0.78 million? Seventy-eight hundredths of a million is a space-saving version of 780,000 human beings. This example is where we must interpret a number across place.

Reasoning across place is equally as important as reasoning within place. Yes, there are 7 tens in the tens place in the number 777. However, bridging between the need to name location and a number's value relationship is what leads to the knowledge that $777 = (7 \times 100) + (7 \times 10) + (7 \times 1)$ as well as 7.77×100, 77.7×10, and 777×1.

Place value through the lens of multiplication

Place value at its core is a multiplicative relationship (see **figs. 1** and **2**). Imagine purchasing supplies for the school year. You are shopping for colored markers for students to use for various projects. You do not buy a single marker. Markers come in boxes. The store stocks markers in boxes of ten. As you select boxes, you have to think simultaneously that for every one box of ten, you get ten individual markers inside that box, a 1:10 box-to-markers relationship. If you buy three times as many boxes, you simultaneously have three times as many individual markers.

These relationships become more sophisticated as the number of markers increases. A shop owner orders markers by the carton. If 10 markers are in a box, 100 boxes are in a carton, and 10 cartons are ordered, then the 10 cartons contain 1,000 boxes, which equates to 10,000 individual markers. These capacities to reason in scale about creating units of units, coordinating the units as they transform, and thinking simultaneously are attributes of place value and the numerical operation of multiplication.



WAVEBREAKMEDIA (LEFT); RANTA IMAGES (RIGHT); GREENWATERMELON (THOUGHT BUBBLES)/THINKSTOCK

A five in the tens place represents fifty, not five.

The role of language

In the classrooms this article describes, a strong position is taken on the consistent use of the language of value that teachers and students use. Numbers are described in value rather than by the digit's name. If adding 54 + 38, regardless of where one starts, the "five in the tens place" is fifty, not five. Thus 50 + 30 = 80, 4 + 8 = 12, 80 + 10 + 2 = 92. All work described below is based on this premise of language usage.

Primary grades

To develop the sense of place value as a rate of ten, Carpenter and his colleagues (2015) have articulated using multistep multiplication and addition problems along with measurement division problems (number of groups are unknown) with ten as the organizing unit to foster children's conceptualization of place value and base-ten understanding. Inspired by this research, Mr. W crafted the following task and posed it to his first-grade students:

> The head soccer coach has 8 bags of soccer balls for all the teams in the league to use. Inside each bag are 10 balls. He also has a bag with only 2 balls in it. How many soccer balls does he have for all the teams?

After spending some time having students visualize the scene, Mr. W moves around the room and visits with various students as they work. Ellamarie is drawing eight rectangles with ten circles inside each rectangle. She then proceeds to count all the circles starting from the number 1. Mr. W asks a few questions about what each rectangle ["I am pretending those are the bags."] and each circle ["I am pretending those are the soccer balls."] are in her drawing.

Amir makes eight trains of ten snap cubes along with two single cubes. Asked about his thinking, Amir states, "These are my bags [*sweeping his hand along one train of cubes*], and these are my soccer balls [*pointing to the individual cubes*]. He proceeds to skip-count the trains by tens to get eighty; then he says, "Eighty-one, eighty-two. Eighty-two soccer balls."

Isabella has the number 82 written on her paper but no evidence of tools or pictures. When asked how she got her answer, she explains, "I know that eight tens are eighty, plus two more makes eighty-two." Mr. W then asks her how many soccer balls the coach would have if he has fourteen bags with ten inside each as well as one bag with only six in it. Isabella is unsure and begins to work on skip counting and making tally marks to track her answer.

By placing ten in each bag, rather than eight, students come to understand that 82 is composed of 8 tens and 2 ones. The range of children's abilities in this classroom includes Ellamarie, who counts out all by ones while placing those into collections of ten, and Amir, who uses trains of cubes to model eight tens and two ones. Both children are direct modelers but demonstrate different understandings of organizing by tens. Isabella quickly answers 82 because she is automatic with a two-digit number, yet she needs to revert to skip counting to determine a three-digit number. After a while, Mr. W brings the class back together and engages in public sharing. Mr. W asks, "Amir, Isabella says that eighty is eight tens. Where in your blocks do you see her answer?" The teacher's questions and prompts allow the children to compare and contrast notions of how place value is constructed in the number 82. Postreflective questions aid students in seeing efficiencies: "Isabella, you skip counted fourteen bags. Now that you know there would be 140 balls, what did you discover when you got to ten bags? (100 balls) And the four bags? (40 balls)

Mrs. T crafted and posed the following task to her first graders:

> The class basket of markers has eighty-four markers in it. At the end of the school year, the teacher wants to place the markers back into boxes. If ten markers fit in a box, how many full boxes will she have?

In this measurement division scenario, children can range from direct modelers to counters to those who are automatic in understanding that 80 is 8 tens, or 8 full boxes. Exploring, modeling, and publicly comparing and contrasting strategies allow children to comprehend the inherent rate of relationships of ten. Such actions bring deeper meaning to CCSSM's call for calculating on the basis of place value and the properties of operations.

Across grades, number size is expanded:

GREENWATERMELON/THINKSTOCK

In the crate are 638 apples that need to be placed in plastic bags and tied shut. If 10 apples go inside each bag, how many full bags can be filled and placed on the shelf for sale to customers?

Although students might come to recognize a pattern to determine the answer, consider the robust level of reasoning by Salli, a third grader, when prompted to explain her thinking: "I know that there are 10 tens in a hundred, so there are 60 tens in 600. There would be 3 tens in 30, so that would be 63 bags of apples."

Salli demonstrates using $10 \times 10 = 100$ as a landmark to reason that if there are six times as many hundreds, there would be six times as many tens. She is reasoning in scale. Using multistep multiplication and addition and measurement division problem types, plus stretching number combinations across grade levels, foster this capacity to comprehend place value as a multiplicative rate of ten. If such experiences and conversations occur across primary grades, then reasoning as a fourth or fifth grader about how many boxes of ten or cartons of a hundred can be generated from packaging 15,425 individual items is built on a progression of ideas rather than simplistic procedures.

Intermediate grades

With the development of multiplication as an operation in third and fourth grade, strengthening one's understanding of place value becomes foundational to understanding the properties of this operation. The following math string develops third and fourth graders' initial idea of scale factor around ten as a unit. Write on the board and ask, "Do you agree with me that there are 10 tens in 100? (Verification necessary.) Use that information to figure out how many tens are in 300." The intent is to have students, over time, progress from skip counting to determining that if I have three times as many hundreds, I will have three times as many tens. As the teacher in this situation, I am drawing out various forms of thinking as students share. For those beginning to think more in scale, I intentionally add arrows to accentuate the linkage for students (see **fig. 3**).

The string can be continued by asking, "If you know 30 tens are in 300, how many tens are in 600?" Here the scale is two times rather than three times larger. A related string can start, "Do

FIGURE 1

At its core, place value is a multiplicative relationship.

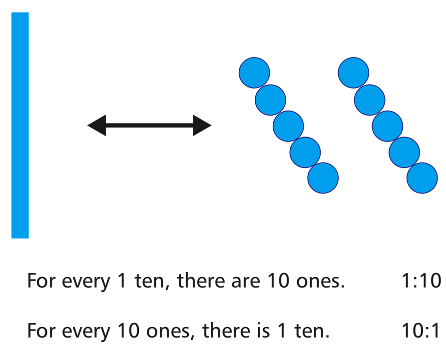


FIGURE 2

This shows the coordination of units and simultaneous thinking.

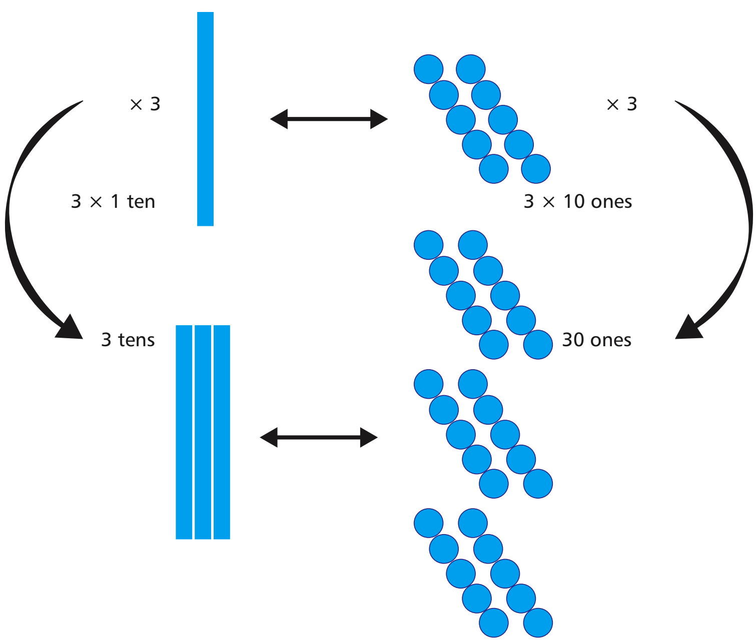
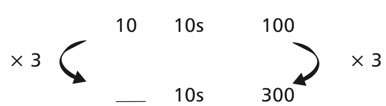


FIGURE 3

This teacher move highlights the scale factor.



I understand that 3×4 is the same as 12, but I thought 12 + 0 was still 12. Isn't a number plus zero the same number you started with?

you agree with me that 10 tens are in 100? How many tens are in 150? The teacher's decision-making point is to stretch students to notice relationships between and among numbers and scale up accordingly. The number combinations can include other starting combinations, such as 5 fives in 25. However, if the focus of concept development is place value and base-ten understanding, then working with units of ten is key to the instructional decision making.

Ms. C. posed 3×40 to her third graders.

Julian answered, "I know 3×4 is 12. I added a zero and got 120."

Ms. C., alert to this surface pattern explanation, responded, "I understand that 3×4 is the same as 12, but I thought 12 + 0 was still 12. Isn't a number plus zero the same number you started with?"

The conversation then centers on how 12 becomes 120 without relying on the surface explanation of counting zeros. In the work done in the classrooms for this project, students are immediately asked to reflect on how 40 is decomposed to allow a person to work with just the 4. The concept of factoring is explicitly introduced, and the associative property is explored. Thus $3 \times 40 = 3 \times (4 \times 10) = (3 \times 4) \times 10$. So, 12 tens ($12 \times 10$) becomes its own opportunity to explore both decomposition of number and using the landmark of 10 tens being 100. I know that 10 tens are 100 and 2 tens are 20, so I get 120. Thirds graders respond to this level of justification if the underlying mathematics is explicitly explored and nurtured.

These conversations about working with powers of ten allow fourth graders to grapple with why a double-digit number times a triple-digit number should be at least in the thousands. Triesha, when multiplying 30×300, decomposed each into factors: "I know 30 is 3×10 and 300 is 3×100; $3 \times 3 = 9$, $10 \times 100 = 1000$, $9 \times 1000 = 9000$. The answer is 9000."

FIGURE 4

Below is a re-representation into an equivalent format.

$$2.6 \times 5.3 = \frac{26}{10} \times \frac{53}{10} = \frac{1{,}378}{100} = 13.78$$

How many hundreds are in the number 1,378?

Had the task been 34×312, Triesha knows her answer is at least above 9,000 as she works with the first partial product and explicitly identifies the factors of ten. Her place-value understanding and concept of factoring is more grounded. She and her fellow students reasoned more mathematically and began to think more multiplicatively as they focused on decomposition of number, explicitly acknowledged the powers of ten, and scaled up while deriving their answers.

This same reasoning around factors of ten extends to the work of fifth graders when working with decimals. For instance, to solve 2.6×5.3, rather than following the procedure of "ignore the decimals, multiply as whole numbers, then count the decimal places starting from the right," students are nurtured to reunitize two and six-tenths ones to be the same as twenty-six tenths ($2.6 \times 1 = 26/10$). Re-representing numbers in fractional form makes the mathematics transparent to students (see **fig. 4**). Notice how the end expression, 1,378/100, cycles back to the primary grades' question of how many hundreds are in the number 1,378. Place value as a rate of ten is reinforced rather than obscured by a procedural surface pattern.

Summary

Place value is more than just naming a digit's value within a particular location. At its most robust level, it is a rate of ten. It is grounded in the conceptual underpinnings of multiplication. Using multistep multiplication and addition as well as measurement division problem types with ten as the organizing unit will focus conversations around these relationships. Describing number by using the language of value matters. Understanding decomposition of number is integral in terms of both a number's addends and factors. Making the mathematics transparent by using varied representations allows students to move beyond surface pattern treatments to truly comprehend the underlying mathematical mechanics. The vertical progression of thinking from one grade level to the next is essential in creating a conversational arc over time so the standards become connected. These are the ingredients for developing a robust understanding of place value.

Common Core Connections

2.NBT 3.NBT
4.NBT.1 5.NBT.1

BIBLIOGRAPHY

Carpenter, Thomas, Elizabeth Fennema, Megan L. Franke, Linda Levi, and Susan B. Empson. 2015. *Children's Mathematics: Cognitively Guided Instruction*. 2nd ed. Portsmouth, NH: Heinemann.

Common Core State Standards Initiative (CCSSI). 2010. Common Core State Standards for Mathematics (CCSSM). Washington, DC: National Governors Association Center for Best Practices and the Council of Chief State School Officers. http://www.corestandards.org/wp-content/uploads/Math_Standards.pdf

Lamon, Susan J. 1996. "The Development of Unitizing: Its Role in Children's Partitioning Strategies." *Journal for Research in Mathematics Education* 27, no. 2 (March): 170–93.

I feel good about my understanding of place value.

James Brickwedde, **jbrickwedde@projectmath.net**, is an assistant professor in the Teacher Education Department at Hamline University and the director of the Project for Elementary Mathematics in St. Paul, Minnesota. He is interested in progressions in students' thinking about place value as they transition from additive to more multiplicative ways of thinking.