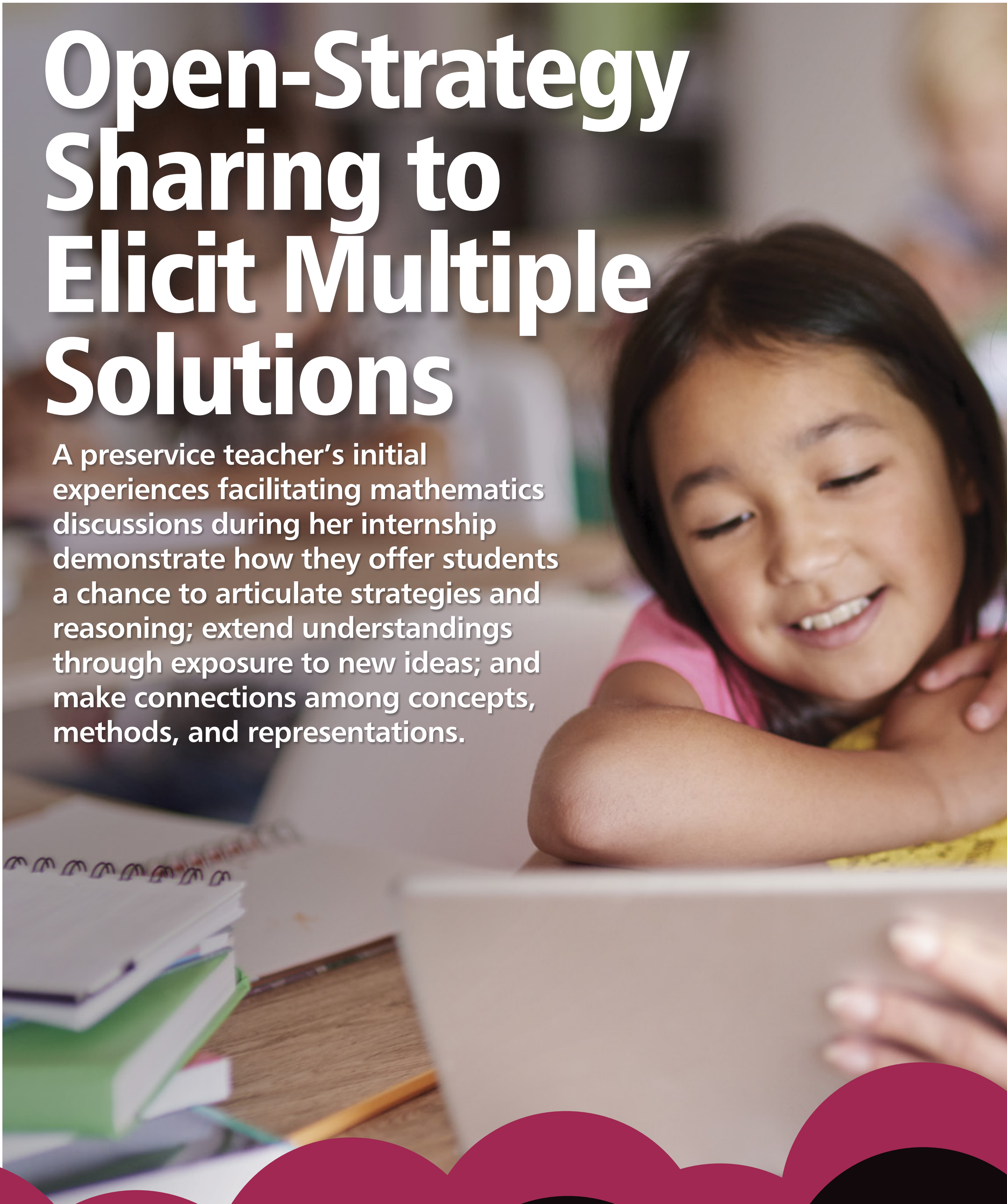
Open-Strategy Sharing to Elicit Multiple Solutions

A preservice teacher's initial experiences facilitating mathematics discussions during her internship demonstrate how they offer students a chance to articulate strategies and reasoning; extend understandings through exposure to new ideas; and make connections among concepts, methods, and representations.



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"Each of you found the solution, thirty and sixty! Are there any *other* solutions?" asked coauthor Hayley Hassinger, a preservice teacher facilitating a mathematics discussion for the first time. All four third-grade students at her table had generated the same solution to a task with multiple solutions. While the students quietly studied their written work, Hassinger wondered to herself, What can I do to elicit more ideas, strategies, and solutions from my students?

Mathematical discussions can provide opportunities for students to articulate strategies and reasoning; extend understandings through exposure to new ideas; and make connections among concepts, methods, and representations. However, as Hassinger's wondering suggests, facilitating productive discussions can be challenging. In this article, we share an account of her initial experiences facilitating mathematical discussions during her student-teaching internship. We hope that learning about the challenges Hassinger faced, the insights she gained, and the planning and reflection tools she used will benefit teachers and teacher educators working to develop effective mathematics instruction.

Open-strategy sharing offers a starting point for teachers who are new to leading mathematical discussions.

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Open-strategy sharing

We focus on one common type of many different classroom discussions, open-strategy sharing (OSS), because it offers a starting point for teachers new to mathematical discussions. The goal of an OSS discussion is to explore multiple possible ways to solve a single mathematical task (Kazemi and Hintz 2014). Students share strategies, listen to others' strategies, and consider relationships among different strategies for solving the same task. To encourage exploration of a wide range of ideas during the discussion, teachers and students can use such *talk moves* as *revoicing*, *repeating*, *reasoning*, *adding on*, *wait time*, *turn and talk*, and *revising* (Chapin, O'Connor, and Anderson 2009; Kazemi and Hintz 2014). **Table 1** shows examples, some from Hassinger's discussions, of each talk move. Through OSS discussions, students learn that "their teachers are interested in their ideas, and with the intentional use of talk moves, they learn what it means to listen and make sense of each other's ideas and to revise their thinking" (Kazemi and Hintz 2014, p. 38).

TABLE 1

Some examples of each talk move are from Hassinger's discussions with her students.

Examples of math talk moves

Talk move	Sample teacher prompt
Revoicing	"I hear you saying that thirty and sixty work because they are thirty apart and their sum is ninety."
Repeating	"Can you repeat the strategy that your partner shared with you?"
Reasoning	"Why does it make sense to try a smaller rather than bigger addend?"
Adding on	"Would anyone like to add on to this idea?"
Wait time	"Take a minute to think about the similarities and differences among the strategies that we came up with."
Turn-and-talk	"Please turn and talk with your partner about the strategies that you used."
Revise	"Did this idea change anyone's thinking about this problem?"

Learning to lead open-strategy sharing discussions

Below we share Hassinger's experiences as she followed the guidelines of a mathematics methods course assignment and conducted two OSS discussions with the support of her third-grade mentor teacher and co-author Katie Roth.

Consulting with a mentor about talk moves

For teachers new to mathematical discussions, consulting with a more experienced colleague for advice and encouragement can be reassuring. Before Hassinger facilitated her first discussion, she and Roth had a conversation using the questions in **figure 1** about the role of talk moves in students' learning of mathematics. We have found these questions to be helpful in guiding initial conversations with colleagues about talk moves.

Roth shared with Hassinger that *adding on* and *turn and talk* were the most natural talk moves for her. *Turn and talk*, when students discuss ideas with a partner before sharing with the larger group, is Roth's favorite talk move in mathematics: "Students are able to share their strategies while gaining a deeper understanding. They can discuss misunderstandings while developing social relationships. And the sharing portion of math talks provides me with feedback about students' understanding of concepts."

Because Hassinger had observed "math buddy talks" first-hand during her time in Roth's classroom, this conversation was particularly meaningful and made Hassinger excited to lead her own discussions.

Preparing to lead an OSS discussion

To plan for her first discussion, Hassinger looked for an open-ended mathematical task that involved concepts that her third-grade students were studying. She and Roth selected this task:

> You add two numbers that are almost 30 apart. The answer is almost 90. What might the numbers be? (Small 2012, p. 27)

Hassinger considered this task (also see **fig. 2**) to have a high level of cognitive demand (Smith and Stein 1998) and strong potential to generate discussion. She explained, "With the way the question is stated, there is no specific procedure to be used, and the word *about* leaves it open to multiple answers." Roth helped Hassinger select four students who would participate actively in a conversation about this task.

Hassinger used prompts (see **fig. 3**) to further prepare for her discussion. Some of these prompts appear in an OSS template developed by Kazemi and Hintz (2014, p. 136; several prompts in **fig. 5** are also drawn from this template.) In our experience, these questions offer useful guidance when preparing for OSS discussions. Hassinger anticipated several strategies students might use:

- Guess and check to find numbers that are *exactly* thirty apart and have a sum of *exactly* ninety
- Guess and check to find numbers that are

FIGURE 1

Teachers might use these questions about the role of talk moves to help guide initial conversations when consulting with a more experienced colleague for advice and encouragement.

Questions about teachers' use of math talk moves

- With what math talk moves are you most familiar?
- How do you use these talk moves in your classroom?
- How do these talk moves support children's learning of mathematics?

FIGURE 2

For her first OSS discussion, Hassinger considered this task to be cognitively demanding, with strong potential to generate discussion because it states no specific procedure "and the word *about* leaves it open to multiple answers."

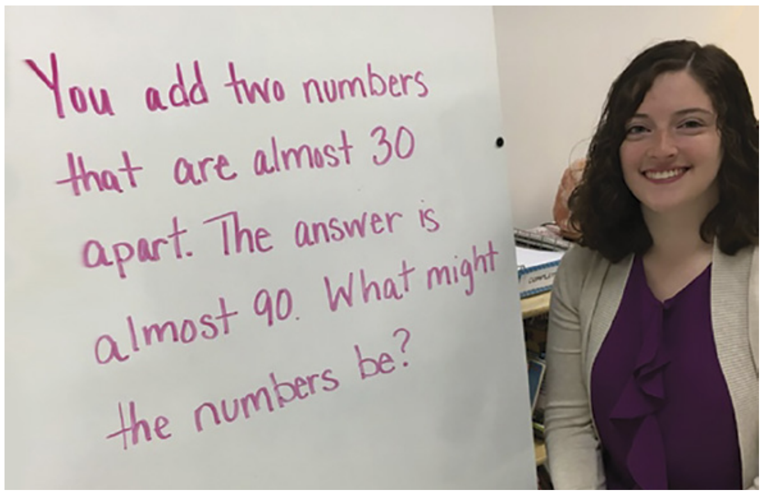


FIGURE 3

Hassinger had observed "math buddy talks" in Roth's classroom. She used these prompts to prepare for initial OSS discussions with four students.

Prompts to guide planning for open-strategy sharing

- What are my plans for opening the discussion?
- How might students solve this problem?
- What materials or tools will be available to children?
- What talk moves can I use to encourage children to share their thinking and to connect with other children's thinking about this task?
- What are some specific focusing questions for this task that I can ask to help me elicit children's thinking about the mathematics?
- What are reminders I can give myself about how to support, rather than take over, children's thinking?
- What are my plans for closing the discussion?

almost thirty apart and have a sum of ninety

- Number lines
- Addition or subtraction

She planned to use *wait time* and have students *turn-and-talk* and *repeat* during the discussion. Hassinger also developed two *focusing questions* to help students clarify and articulate their ideas (Herbel-Eisenmann and Breyfogle 2005):

1. How might you prove that _____ and _____ are your solutions to this problem?
2. What do you notice about the solutions we found? What are the similarities and differences?

Because she aimed to elicit students' thinking as much as possible, Hassinger reminded herself, Do not dominate the conversation. Students should be doing the talking and thinking.

Leading an initial OSS discussion

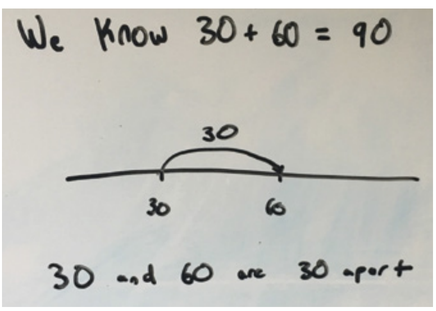
Hassinger began her discussion by reading the task—which was written on a whiteboard—aloud. Although she was tempted to give students hints about the task, Hassinger refrained because she "wanted to see how they made sense of the problem on their own." She reread the task multiple times as students worked individually for several minutes. Whenever she read the task aloud, she placed emphasis on the word *almost*.

Students shared their strategies with partners using *turn and talk*. Then, because Hassinger thought *repeating* would help students make connections across strategies, she asked each student to explain his or her partner's strategy. As the students did so, Hassinger noticed that all four students identified the number pair 30 and 60 as a solution—consistent with one of the strategies she had anticipated. Although all students identified this solution, they illustrated their strategies differently. For example, one student used two equations (30 + __ = 90 and 30 + 30 = 60) to explain that thirty and sixty have a sum of ninety *and* that thirty and sixty are thirty apart. Another student succinctly stated that she used *guess and check* to find that thirty and sixty are thirty apart and that their sum is ninety.

Because Hassinger hoped to have students generate solutions that take into account the word *about* in the task, she asked the students several times to find another solution. However, the students did not, and she realized she could not use her second focusing question about comparing solutions. At a point when the discussion felt "flat" to her, Hassinger decided to ask about number lines—a representation that one student had used but which had not been shared with the group. She wondered aloud if a number line could be used as a tool for solving this task, and two students proceeded to represent their original solutions on number lines (see **fig. 4**).

FIGURE 4

A representation in Hassinger's first math talk shows how a student used a number line to demonstrate that the numbers 30 and 60 have a difference of 30.



Reflecting and revising

The prompts in **figure 5** can help teachers reflect on an OSS discussion and identify possible areas for improvement. One prompt involves looking for indicators that the teacher may have inadvertently solved part of the task for students or dominated the conversation (Jacobs et al. 2014). Although Hassinger had hoped students would explore multiple strategies and solutions in the discussion, she felt instead that with “little variation in the students’ responses, our discussion dissipated quickly.” Hassinger was curious about why each student responded with thirty and sixty, rather than solutions in which the numbers have a difference of *almost* thirty and a sum of *almost* ninety. She speculated that if students had attended to the “almost” parts of the task, more variation among solutions and deeper discussion might have developed. Hassinger wondered if, as she read the task aloud multiple times at the beginning of the discussion, she had perhaps interrupted students’ initial thinking about the task rather than helping them to engage with it.

As she prepared for her second OSS discussion with a new group of students, Hassinger made several changes to her plans. Whereas she had not helped the first group of students to interpret the task, she planned to begin her second discussion by asking students to consider the meanings of the words *about* and *estimating*. She also created a new version of the task:

> You add two numbers that are almost 24 apart. The answer is almost 83. What might the numbers be?

Hassinger hoped that because this task contains “numbers that are not multiples of five or ten,” students would estimate and generate multiple solutions. Additionally, she made changes to the talk moves she planned to use. Instead of having partners *repeat* each other’s solutions, she planned to ask for further elaborations of selected strategies.

Leading a second OSS discussion

Hassinger began her second discussion by asking students about their prior experiences with estimation and trial and error. She felt that this prepared students to engage with the revised task. After reading the task aloud, she encouraged students, “If you find one pair that works, see if you can find another pair.”

Hassinger remarked on the numerous solutions that emerged from this second discussion: “Students really used their number sense to determine what numbers were possible. They recognized certain numbers would not work because the number would be too small or too large compared to eighty-three.”

FIGURE 5

Reflection on an open-strategy sharing discussion can help teachers improve in leading the next discussion.

Questions for reflection after an OSS discussion

- Looking back at the anticipated strategies, who solved it this way?
- What other strategies emerged during the discussion? Who solved it this way?
- On the basis of this mathematical discussion, what questions do I now have about my students’ understandings?
- What did I notice, in watching my video, about the three “warning signs” (interrupting the child’s strategies, manipulating the tools, or asking a series of closed questions)? How might I address those in my next discussion?
- To what extent did the discussion provide opportunities for each student to share his or her thinking about the mathematics?
- To what extent did the discussion provide opportunities for students to engage with one another’s mathematical ideas?

FIGURE 6

A student used a solution with equations and a number line, explaining, “I did twenty-six and fifty-three, and it was twenty-seven apart and when added together equaled seventy-nine.”

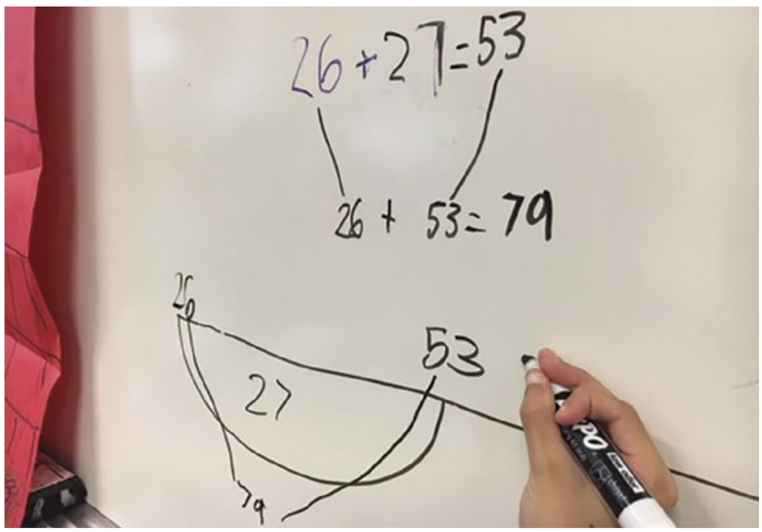
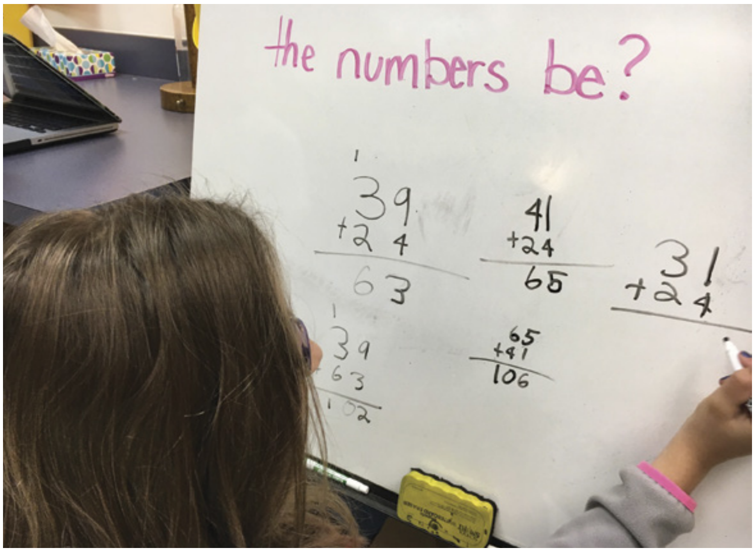


FIGURE 7

This student is using a trial-and-error strategy to get closer to a sum of 83.



One interesting solution that the group discussed was the number pair 30 and 50, which a student found by working with tens. Hassinger asked probing questions of the group to explore this solution:

***Hassinger*:** How can you prove that fifty and thirty works as a solution?

***Girl*:** Fifty plus thirty is eighty, and that's close to eighty-three.

***Hassinger*:** How can you prove that fifty plus thirty is *almost* eighty-three?

***Boy*:** You could draw a number line.

***Hassinger*:** Let's do that. Let's draw a number line to show that your numbers are *almost* twenty-four [numbers] apart and equal *almost* eighty-three.

After the group explored this solution with a number line, each student used number lines to find or represent other solutions with differences close to twenty-four. One boy, who initially represented his work with equations, drew a number line to illustrate a difference of twenty-seven (see **fig. 6**). He explained, "I did twenty-six and fifty-three, and it was twenty-seven apart and when added together equaled seventy-nine."

Identifying pairs of numbers that satisfy the conditions of this task took a significant amount of work for students, and they were eager to share their thinking. One student used an addition strategy to find a pair of numbers, 39 and 63, with a difference of 24 and a sum of 102 (see **fig. 7**). To generate a sum closer to 83, she tried changing 39 to 41. This increase resulted in a sum that was farther from, not closer to, her goal. For her third pair, she decreased her starting number and found a solution, 31 and 55, with a sum close to 83. From here, she adjusted her differences so that they were "almost" 24 rather than "exactly" 24 to find a complete solution to the task.

Toward the end of the discussion, Hassinger asked, "Do we notice any similarities or differences or any patterns when looking at our final solutions?"

A student observed that in choosing number pairs, "We all started in the twenties or thirties."

Another student suggested that by making small adjustments to the numbers in a solution pair, new solutions could often be generated. Hassinger's focusing question provided an opportunity for students to revisit and articulate diverse processes that they used while searching for solutions.

Looking back

Reflecting about her two OSS discussions, Hassinger commented that "the biggest difference was the solutions that my students came up with." Whereas the first group of students generated a single solution, in the second group, "no one's solutions were the same." Hassinger attributed this difference to her introductory discussion and the change in the task: "I noticed that the task and the lead-up questions were crucial to students' understanding." She also described feeling more relaxed and better prepared to ask probing questions in the second discussion. She intentionally asked more "why" questions that prompted students to explore strategies in depth.

Conclusions

Hassinger improved her OSS discussions by making deliberate changes to the mathematical task and the way she prepared students for the task. Her second group of students shared and explored multiple strategies and solutions, as is the goal of an OSS discussion. Other teachers can draw on this preservice teacher's experience to recognize that careful planning, focused reflection, and purposeful experimentation can contribute to positive experiences with mathematical discussions.

As preservice and practicing teachers work to incorporate OSS discussions into their instructional repertoires, they may appreciate the support of such tools as the prompts that Hassinger used. These prompts offer a helpful structure for planning and reflection as well as for making comparisons across discussions. Teacher leaders can use these tools and others (e.g., videos of discussions, student work) to support teachers' ongoing learning about OSS discussions. Moreover, teachers and teacher leaders can collaborate on the improvement of mathematical discussions. For example, in the case of Hassinger's discussions, a collegial group could first learn about Hassinger's decision making as she made adaptations and then notice the impact of those changes on students' mathematical contributions. Colleagues might also generate additional questions for Hassinger to ask students (e.g., Do the numbers 30 and 60 fully satisfy the conditions in the task? How can we check?). By improving teachers' capacity to lead rich discussions, we increase opportunities for students to develop, share, and extend their mathematical understandings.

Common Core Connections

3.NBT.2
SMP 1
SMP 3

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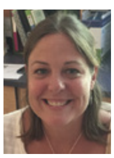

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